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Emission regulation: Prices, quantities and hybrids with endogenous technology choice

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Abstract

This paper examines the investment incentives of market-based regulation, with focus on the technology characteristics the different regulatory schemes tend to incentivize. The firms' technology choice is socially optimal if and only if the aggregate emission allowance supply is completely inelastic. Further, in the presence of uncertainty, elastic emission allowance supply and strictly convex environmental damage, it is optimal to tax investment in technologies that induce large variance in emissions. Last, price elastic supply of emission allowances may increase the volatility in the product market, depending on the risk environment the firms face. The results indicate that introduction of permit price stabilizing measures in an emission trading system will come at the cost of suboptimal technology investments. It may also cause increased fluctuations in product prices.

Keywords: Regulation; technology choice; uncertainty; investment; welfare.

JEL classification: Q52; Q58; D81; H41.

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Sammendrag

Denne artikkelen undersøker hvilke typer teknologi ulike typer markedsbasert regulering av utslipp gir insentiver til å investere i.

Bedriftenes teknologivalg er optimalt fra et samfunnsøkonomisk perspektiv hvis og bare hvis det samlede totale utslippt er eksogent gitt. Det betyr at innføring eller utvidelse av ordninger der antallet utslippskvoter kan tilpasses ut fra etterspørselen etter kvoter, som f.eks. markedsstabilitetsreserven (MSR) i EU sitt kvotesystem, kan gi insentiver til investering i teknologi som er mindre hensiktsmessig fra et samfunnsmessig ståsted.

Videre kan det være gunstig å skattlegge investeringer i teknologi som fører til relativt høye svingninger i utslippene, gitt endogene totale utslipp og usikkerhet rundt produksjonskostnader eller etterspørselen etter godene som gir utslipp. Til slutt kan et system der antallet utslippskvoter påvirkes av etterspørselen etter kvoter øke volatiliteten i produktmarkedet. Dette avhenger imidlertid av typen usikkerhet bedrifter og konsumenter står overfor.

Resultatene indikerer at innføring av prisstabiliserende tiltak på utslippskvoter kan gi suboptimale teknologiinvesteringer. Det kan også føre til økte svingninger i produktprisene.

1 Introduction

Implementation and development of new technologies have been essential for ameliorating various environmental challenges such as acid rain, climate change and depletion of the ozone layer. In the longer term, the ability to drive investment and research and development is therefore arguably among the most important criteria when evaluating environmental policy (See, e.g., Kneese and Schultze (1975), Orr (1976) for and Jaffe and Stavins, 1995 for early presentations of this view).

It is therefore not surprising that a substantial economic literature examines environmental policy and the implementation of new technology (see Jaffe et al., 2002, Löschel, 2002, or Requate 2005 for literature reviews). One branch of this literature focuses on the investment levels the various regulatory schemes spur, often with a focus on prices versus quantities-based regulation (see, e.g., Denicolo, 1999, and Requate and Unold, 2003). Another part of the literature examines various market failures and rationales for subsidizing technology investment, typically with a focus on subsidies motivated by positive externalities such as technology spillovers or learning by doing (see Arrow (1962) for a seminal paper on learning by doing. See Golombek and Hoel (2005) and Kverndokk and Rosendahl (2007) for its' implications for environmental policy).

In this paper, the focus is not so much on the total *level* of investment that various types of regulation provide incentives for, but rather on the *type* of technology that is promoted. This addresses a shortcoming of the literature first pointed out by Krysiak (2008), i.e., the literature tends to analyze the amount of investment, but do not consider what type of technology it is invested in. In addition, this paper investigates the rationale for active technology policies that involves taxing investments in certain technology characteristics. More precisely I show that, in the presence of uncertainty, endogenous aggregate emissions and strictly convex environmental damage, it is optimal to tax investment in technologies that induce large variance in emissions. The reason is that fluctuations in emissions creates a negative externality that is not internalized by the firms. The fact that these fluctuations are influenced by the firms' technology choice motivates active technology investment policies. Perhaps somewhat counterintuitive, optimal policy may thus involve taxation of investment in certain types of abatement technology.

Emission trading and emission taxes are presently by far the most prominent examples of implemented price or quantity based regulatory policies. The most well-known example of carbon ETS today is perhaps the European Emission Trading Scheme (EU ETS), but the number of ETS is growing fast and currently includes schemes in, e.g., China, Australia, Canada, New Zealand and Japan. Emission trading is also used to reduce other pollutants, such as the U.S. NOx Budget Trading Program (nitrogen oxides) and the U.S. Acid Rain program (sulphur dioxide). Whereas there are currently several international emission trading schemes, emission taxes are predominantly used nationally. Important examples are the U.S. tax on ozone depleting chemicals and taxes on CO2 emissions in several European countries.[1]

As pointed out by Grull and Taschini (2011), policy makers tend to have different views about the range of acceptable prices for tradable emission permits. Hence, there has been an interest in mechanisms to keep the permit price from rising too high or falling too low. One important example of such permit price stabilizing mechanisms is the EU ETS Market Stability Reserve (MSR), which aims to absorb historical surplus of allowances and ensure market stability.[2] The idea of creating a hybrid system by combining emission trading and taxes was first introduced by Roberts and Spence (1976), see Hepburn (2006) for an overview of the literature.

In this paper, I investigate a potentially unwanted side-effect of elastic supply of emission allowances. That is, price elastic supply of emission allowances influences the volatility in the product prices. For example, the volatility in the price on electricity in the area covered by the EU ETS will be affected by emission allowance stabilizing measures such as the Market Stability Reserve (MSR). Whether the electricity price volatility increases or decreases depends on the characteristics of the demand and supply-side uncertainties. More precisely, price elastic supply of emission allowances involves increased volatility in electricity prices if the shocks to electricity demand and electricity generation costs together are sufficiently large as compared with the shock to abatement costs in electricity generation.

Weitzman (1974) examined regulation of public goods in the presence of uncertainty. Whereas price-based regulatory instruments fix the price of licenses, but leave the issued quantity uncertain, quantity-based instruments fix the quantity of licenses issued but leave the price uncertain. This trade-off raises the following question for policy design: which type of regulation best help mitigate the cost of uncertainty so as to maximize social benefits of the public good? Weitzman (1974) found that price-based instruments are advantageous when the marginal benefit schedule is relatively flat as compared to the marginal cost schedule, and vice versa. This has since been the consensus among most economists (e.g., Kolstad, 2000; Hoel and Karp, 2001; Pizer,

[1] See https://www.c2es.org/content/cap-and-trade-basics/ for a map over regional, national and subnational carbon pricing initiatives implemented, scheduled for implementation and under consideration (ETS and carbon tax).

[2] Every year, by 15 May, the EU commission publishes the total number of emission allowances in circulation (TNAC). Allowances are then transferred to the MSR if the TNAC exceeds the upper threshold. In the period 2019-2023, the upper threshold value is 833 million allowances in circulation and the transfer rate is 24 per cent. Similarly, allowances are released from the MSR to the market if the TNAC is below the lower threshold value. In the period 2019-2023, 100 million emission allowances were to be released if the TNAC fell below 400 million allowances. Note that a high TNAC implies large supply relative to the demand of emission allowances, which yields a low allowance price. Hence, the MSR involves increased (reduced) supply if the allowance price is high (low), as also stipulated by equation (3). Unlike equation (3), the MSR is only active if the threshold values are crossed, however. See Perino and Willner (2016), Chaton et al. (2018), Brunix et al. (2020) and Reyer et al. (2020) for more on the MSR. See also https://ec.europa.eu/commission/presscorner/detail/en/qanda_21_3542

2002; Nordhaus, 2007). The results in the present paper corroborates the results in (Krysiak (2008) and Storrøsten (2014; 2015) that, as compared with Weitzman (1974), endogenous technology choice provides a comparative advantage for quantities over price-based regulation. It also extends the result to hybrid regulation with price elastic permit supply and more general uncertainty and cost structures. In particular, the competitive firms' technology choices remain optimal under (standard) emissions trading also when the firms can choose their production technology (and not only abatement technology as in Krysiak (2008) and Storrøsten (2014; 2015)).

Stigler (1939) and Marschak and Nelson (1962) early examined firms' choice of cost structure and referred to the firms' ability to change production levels in response to new information as their "flexibility". Mills (1984) continued this terminology and showed that an unregulated competitive firm will invest more in production flexibility if demand uncertainty increases. Mendelsohn (1984) examined investment in a prices vs. quantities setting, and showed that quantity-based instruments have an advantage, because price-based regulation induces excessive variation in output. Krysiak (2008) shows that technology choice is socially optimal under quantities, but not so under prices. Further, price-based regulation induces a more flexible technology than tradable quantities. Storrøsten (2014) shows that tradable emissions permits and an emissions tax are no longer equivalent when cost structure is endogenous and the product market is taken into account. Storrøsten (2015) derives a criterion comparing prices vs. quantities in terms of expected welfare, given uncertainty, optimal policy and endogenous cost structure.

The present paper adds to the literature on regulation and endogenous technology choice by (*i*) examining the role of technology policies in the presence of uncertainty and endogenous cost structure, (*ii*) expanding the analysis to hybrid schemes with price elastic supply of emission allowances, including the optimal hybrid policy in the presence of endogenous technology, (*iii*) increasing the focus on the product market (of which production causes emissions as a by-product) with uncertain production costs, and (*iv*) allowing for a more general menu of cost structures than previous studies.

Sections 2.1 to 2.3 features the analytical analysis, whereas Section 2.4 presents a brief discussion that puts the results into the well-known analytical framework of Weitzman (1974). Section 3 concludes.

2 Theoretical analysis

The theory model is divided into three stages. First, the regulator determines the regulatory regime in Stage 1. Then, the firms invest in production and abatement technology in Stage 2.

Last, consumers choose consumption and firms choose production and emission levels in Stage 3. The model is solved backwards to find the subgame perfect Nash equilibrium.

Let there be $I = \{1, 2, ..., n\}$ competitive firms and $J = \{1, 2, ..., m, \}$ price taking consumers. The cost function of firm $i \in I$ is:

$$c(q_i, a_i; \mathbf{x}_i, \eta_i, \varphi_i) = (\alpha_i + \eta_i) q_i + \frac{\beta_i}{2} (q_i - \widetilde{q}_i)^2 + (\delta_i + \varphi_i) a_i + \frac{\gamma_i}{2} (a_i - \widetilde{a}_i)^2, \tag{1}$$

where $\mathbf{x}_i = \{\alpha_i, \beta_i, \delta_i, \gamma_i, \widetilde{q}_i, \widetilde{a}_i\}$ are positive technology parameters, q_i is production, a_i is abatement, and η_i and φ_i are firm specific stochastic variables with expected values equal to 0 and variances σ_η^2 and σ_φ^2, respectively. That is, we have $\eta_i \sim \left(0, \sigma_{\eta_i}^2\right)$ and $\varphi_i \sim \left(0, \sigma_{\varphi_i}^2\right)$. Note that this cost structure implies that firms produce at minimum efficient scale (MES) when $q_i = \widetilde{q}_i$. Further, the MES unit cost of production is $\alpha_i + \eta_i$, while the higher cost following $q_i \neq \widetilde{q}_i$ increases in the technology parameter β_i. The interpretation regarding abatement cost is similar.

The stochastic elements may reflect, e.g., fluctuations in input prices or productivity, or a breakdown of equipment. As pointed out by Weitzman (1974), the stochastic variables may stem from genuine randomness or just imperfect information. The stochastic shocks enter the functional form linearly, which is similar to Weitzman (1974), Hoel and Karp (2002), Krysiak (2008) and Storrøsten (2014; 2015).[3]

Firm i's emissions are given by $\varepsilon_i = kq_i - a_i$, where k is a positive constant, kq_i is emissions without regulation (business as usual) and $a_i \leq kq_i$. Note that this stylized specification corresponds to end-of-pipe abatement. The functional form (1) is chosen primarily because it allows for interpretable analytical results. The firm pays $\tau\varepsilon_i$ for emissions ε_i, where τ is the price on emissions.

The firms can choose the technology parameters $\mathbf{x}_i$ in the first stage of the game. The investment cost function, $\kappa(\mathbf{x}_i)$, is convex and decreasing in α_i, β_i, δ_i and γ_i, and satisfies $\kappa(\cdot) \to \infty$ as $\alpha_i\beta_i\delta_i\gamma_i \to 0$. Further, the investment cost function is convex and increasing in the minimum efficient scale parameters $\widetilde{q}_i$ and $\widetilde{a}_i$.

The price on the homogenous good sold by the firms is p. Firm i's profit in Stage 3 is given by:

$$\pi_i = \max_{q_i, a_i} \left[pq_i - c(q_i, a_i; \mathbf{x}_i, \eta_i, \varphi_i) - \tau (kq_i - a_i)\right]. \tag{2}$$

where $c(\cdot)$ is given by equation (1). Note that it is mathematically equivalent whether the firms maximize with respect to emission or abatement levels.

In this paper I examine standard emissions trading (ETS), an emissions tax (TAX), and

[3] The framework in Krysiak (2008) does not feature a product market. Storrøsten (2014; 2015) limits the random elements to abatement costs only.

a hybrid scheme (HYB) where the supply of emission allowances increases in the price on allowances. This is modelled using the following emission allowance supply function:

$$\varepsilon^s = S + v\left(\tau - \widetilde{\tau}\right), \tag{3}$$

where $v \geq 0$, $\widetilde{\tau}$ is the emissions price target and $S = \sum_{i \in I} \varepsilon_i$ is the aggregate emissions target. I assume that these parameters (i.e., v, $\widetilde{\tau}$ and S) are determined by the regulator in Stage 1. Hence they are exogenous in stages 2 and 3. One special case, particularly relevant if moving from ETS with $v = 0$ to hybrid regulation HYB, is where S is the fixed emissions cap and $\widetilde{\tau}$ is the expected permit price given this cap. Even tough v is the slope of the permit supply function, I will sometimes refer to v as the price elasticity of permit supply for convenience (this elasticity is actually given by $v\widetilde{\tau}/\varepsilon$). Equation (3) equals standard emissions trading if $v = 0$, converges towards a standard emissions tax if $v \to \infty$, and a finite (positive) v yields a hybrid scheme (see also Lemma 1 below). Equation (3) determines the emission cap as a linear function of S and τ. Note that this simple scheme does not allow regulation where the price on emissions follows marginal environmental damage, unless the environmental damage function is linear.[4] We also observe that HYB encompasses ETS and TAX. Hence, expected welfare will always be higher or equal under HYB as compared with TAX and ETS (given that the value of v maximizes expected welfare).

The utility function of consumer $j \in J$ is given by:

$$u_j\left(q_j, \theta_j\right) = \left(b + \theta_j\right) q_j - \frac{d}{2} q_j^2 - g_j\left(\varepsilon\right), \tag{4}$$

where b and d are positive parameters, $g_j\left(\varepsilon\right)$ is harm from aggregate emissions, $\varepsilon = \sum_i \varepsilon_i$, and $\theta_j \sim \left(0, \sigma_{\theta_i}^2\right)$ is a consumer specific stochastic variable.

I assume that the outcomes of the stochastic variables η_i, φ_i and θ_j are determined in between stages 2 and 3. Hence, the firms invest under uncertainty in Stage 2, but know their cost and the equilibrium prices when they make the production and emission decisions in Stage 3. The consumers only act in Stage 3, when they choose their consumption levels under full information. This implies that the firms are all equal in Stage 2, because they face the same uncertainty, whereas firms and consumers are heterogenous in Stage 3. It follows that, for each regulatory regime, all firms invest in the same technology. That is, we have $\mathbf{x}_i = \mathbf{x}^g$ where superscript $g = \{ets, hyb, tax\}$ denotes the regulatory regime (see also Appendix A). I henceforth omit the firm specific subscript on the technology parameters.

[4] The price on emissions is equal to marginal environmental damage if $v = \left(\varepsilon^s - S\right) / \left(g_2 \varepsilon^s - \widetilde{\tau}\right)$ in the case with quadratic environmental damage from emissions given by $g\left(\varepsilon\right) = g_1 \varepsilon + g_2 \varepsilon / 2$ (with $g_1 > 0$ and $g_2 > 0$), but I will assume that v is constant and independent from aggregate emissions in this paper.

2.1 Stage 3: Production and consumption

Firm $i \in I$ maximizes profits in Stage 3 and solves (2), given equation (3) and the technology parameters $\mathbf{x}$. Each firm's profit is a sum of linear and concave functions (cf., equation (2)), implying that the objective function in equation (2) is concave. It follows that the maximization problem (2) has a unique global maximum. The industry supply function and demand for emissions can be derived from the first order conditions associated with (2). Aggregate supply and aggregate demand for emissions are given by:

$$q^s = \sum_{i\in I} q_i = \frac{n}{\beta}\left(p - \alpha - k\tau + \beta\widetilde{q}\right) - \frac{\eta}{\beta}, \tag{5}$$

$$\varepsilon^d = \sum_{i\in I} \varepsilon_i = n\left(\widetilde{q}k - \widetilde{a} - \frac{\tau-\delta}{\gamma} + k\frac{p-k\tau-\alpha}{\beta}\right) + \frac{1}{\gamma}\varphi - \frac{k}{\beta}\eta, \tag{6}$$

respectively, with $\eta \equiv \sum_{i\in I}\eta_i$ and $\varphi \equiv \sum_{i\in I}\varphi_i$.

Consumer $j \in J$ solves $\max_q u_j(q_j, \theta_j) - pq_j$, with solution $q_j = \frac{1}{d}(b - p + \theta_j)$.[5] Total demand for the good q is then given by:

$$q^d = \sum_{j\in J} q_j^d = \frac{m}{d}(b-p) + \frac{\theta}{d}, \tag{7}$$

with $\theta \equiv \sum_{j\in J}\theta_j$. The economy is assumed to be closed and hence we have the following product market equilibrium condition:

$$q^d = q^s. \tag{8}$$

The competitive equilibrium solves equations (5), (6) and (7) subject to the market equilibrium conditions (3) and (8).

We see from equations (5), (6) and (7) that the stochastic properties of the aggregate shocks η, φ and θ, and their relations with the individual shocks η_i, φ_i and θ_i, are important for the characteristics of the competitive equilibrium. In this paper I assume that the shocks θ_j, φ_i and η_i are independent, i.e., we have expected values $E[\theta_j\varphi_i] = E[\theta_j\eta_i] = E[\varphi_i\eta_i] = 0$ for all $i \in I$ and $j \in J$. I further assume symmetrically correlated shocks, with correlation coefficients given by $\rho_\eta = E[\eta_i\eta_{i'}]/\sigma^2_{\eta_i}$ and $\rho_\eta = E[\varphi_i\varphi_{i'}]/\sigma^2_{\varphi_i}$ for all firms $i, i' \in I$ $(i \neq i')$ and $\rho_\theta = E\left[\theta_j\theta_{j'}\right]/\sigma^2_{\theta_j}$ for all consumers $j, j' \in J$ $(j \neq j')$. These assumptions are helpful for achieving interpretable theoretical results. Whereas the nature of the shocks is not modelled explicitly, it is reasonable to assume that the correlation coefficients will depend on the origin

[5] The alternate model formulation where the consumer maximizes utility subject to a budget constraint leads to a system of second order equations with solutions for q^s and ε^d that do not permit a tractable analytical solution for the whole model (including technology investment in period 2).

of the uncertainty. For example, shocks caused by fluctuating factor prices will typically be stronger correlated than randomness caused by equipment failure. Note that symmetrically correlated shocks implies that we must have $\rho_\theta \in [-1/(m-1), 1]$, $\rho_\varphi \in [-1/(n-1), 1]$ and $\rho_\eta \in [-1/(n-1), 1]$ for the covariance matrixes to be valid (i.e., positive semi-definite). It can be shown that we have $\sigma^2_{\eta_i} = E\left[\eta_i^2\right]$, $cov(\eta_i, \eta_{i'}) = \rho_\eta \sigma^2_{\eta_i}$, $var(\eta) = n\left(1 + (n-1)\rho_\eta\right)\sigma^2_{\eta_i} \equiv \sigma^2_\eta$, $cov(\eta, \eta_i) = \left(1 + (n-1)\rho_\eta\right)\sigma^2_{\eta_i}$ and $cor(\eta, \eta_i) = \sqrt{\rho_\eta (n-1) + 1}/\sqrt{n}$ for $i, i' \in I$ and $i \neq i'$, with analogous expressions for the variances, covariances and correlations of the other two shocks φ_i and θ_j. Furthermore, we have $var(\eta) \in \left[0, n^2\sigma^2_{\eta_i}\right]$ and $cor(\eta, \eta_i) \in [0, 1]$, where these expressions reach their lower and upper limits at $\rho_\eta = -1/(n-1)$ and $\rho_\eta = 1$, respectively. The particular case with uncorrelated η_i's, characterized by $\rho_\eta = 0$, yields $var(\eta) = n\sigma^2_{\eta_i}$ and $cor(\eta, \eta_i) = 1/\sqrt{n}$ (and similarly for φ_i and θ_j).

The solutions for the market equilibrium product and emission prices are given by:

$$\begin{pmatrix} p^{ets} & \tau^{ets} \\ p^{hyb} & \tau^{hyb} \\ p^{tax} & \tau^{tax} \end{pmatrix} = \begin{pmatrix} E(p^{ets}) + \Psi^{ets}_p(\eta, \varphi, \theta) & E(\tau^{ets}) + \Psi^{ets}_\tau(\eta, \varphi, \theta) \\ E(p^{hyb}) + \Psi^{hyb}_p(\eta, \varphi, \theta) & E(\tau^{hyb}) + \Psi^{hyb}_\tau(\eta, \varphi, \theta) \\ E(p^{tax}) + \Psi^{tax}_p(\eta, \theta) & E(\tau^{tax}) \end{pmatrix}, \tag{9}$$

where the expectations are the non-stochastic parts and the Ψ's are the random elements, see Appendix A for the exact expressions and derivations. Whereas the prices depend on all the three stochastic elements under ETS and HYB, the product price under TAX does not depend on the shocks to abatement costs.[6] Moreover, aggregate emissions are exogenous under ETS, while the permit price is exogenous under TAX (cf., equation 3). The expected values depend on the technology parameters $\mathbf{x}$ and the utility function parameters b and d (see equation (4))

Suppose the emission price target (or tax) is set such that $\widetilde{\tau} = E(\tau^{ets})$ in equation (3), i.e., the expected price on emissions is equal across all schemes. Further, let χ^{ets}, χ^{tax}, χ^{hyb} denote an endogenous variable in the case of ETS ($v = 0$), TAX ($v \to \infty$) and HYB ($0 < v < \epsilon$) for some finite constant ϵ, respectively (e.g., $\chi^{ets} \in \left\{q^{ets}, \varepsilon^{ets}, p^{ets}, \tau^{ets}\right\}$). We then have the following result regarding the relation between HYB, TAX and ETS:

Lemma 1. *Suppose* $\widetilde{\tau} = E(\tau^{ets})$ and *technology* $\mathbf{x}$ *is fixed and equal for all* v. *Then, the endogenous variables satisfy the following:*

(i) $E(\chi^{ets}) = E(\chi^{hyb}) = E(\chi^{tax})$.

(ii) $\chi^{hyb}(v = 0) = \chi^{ets}$ *and* $\chi^{hyb}(v \to \infty) = \chi^{tax}$.

[6] There is no direct effect from shocks to abatement costs to the product market, because the price on emissions is fixed. But without the end-of-pipe assumption in equation (1), there would be indirect effects from abatement levels to production costs via the cross derivatives of the cost function. Note also that there is no exit or entry in this analysis.

(iii) $\chi^{tax} \leq (\geq) \chi^{hyb} \leq (\geq) \chi^{ets}$ *for* $v \geq 0$.

(iv) χ^{hyb} *is strictly monotonic in* v.

Proof. *See Appendix A.*

The first bullet point (i) states that the expected values of the endogenous variables are all equal across the regulatory regimes. The realized prices and quantities will differ, depending on the stochastic shocks, however. Bullet point (ii) simply states that hybrid regulation equals emissions trading if $v = 0$, and converges towards an emission tax as $v \to \infty$ (as pointed out in the discussion of equation (3) above). The third bullet point (iii) states that the solutions under hybrid regulation always is somewhere in between the solutions for standard emission trading and an emission tax. This may not be surprising, as HYB is constructed as a linear combination of ETS and TAX (cf., equation (3)). Last, point (iv) states that χ^{hyb} is either strictly decreasing or strictly increasing in the price elasticity of emission allowances v. So, for example, production and emissions under hybrid regulation will always be in between the outcomes under a tax and standard emissions trading, given equal technology vector $\mathbf{x}$ and the outcomes of the stochastic variables. Further, the regulator can choose to increase v to get closer to emission tax regulation, or reduce v to get closer to emissions trading. The assumption of equal technology across regulatory regimes is crucial in the derivation of Lemma 1, and it does not apply to case of endogenous technology in general. The exception is Lemma 1 (ii), which follows directly from equation (3) and hence remains valid for endogenous technology.

The reduced form solutions in equation (9) allow for comparison of how the different stochastic variables affect the volatility of prices and quantities under the different regulatory schemes. Whereas it is rather intuitive that the volatility in the price on emissions (τ) decreases in v, and that the volatility in the aggregate emissions increases in v (see equation (3) and Appendix A), the results regarding the product market are less obvious. We have the following result:

Lemma 2. *The variances in aggregate production and the product price under TAX are larger (smaller) than under ETS unless* $\sigma_\eta^2 + \sigma_\theta^2$ *is sufficiently small (large) as compared to* σ_φ^2.

Proof. *See Appendix A.*

Lemma 2 states that shocks to production costs (η) and consumer utility (θ) have smaller impact on the product price under emissions trading than under emissions taxes, while shocks to abatement costs (φ) have lower impact under the emissions tax. Regarding η and production, the economic rationale is that the equilibrium allowance price decreases in the shocks to production costs, because the demand for emission allowances decreases when production costs increase and vice versa. That is, the positive shock to costs is partly offset by a lower price on emissions under ETS. This counteracts the initial effect of shocks to production costs (η) under emissions trading. Similarly, the shock to demand θ also has a smaller impact on aggregate production under emissions trading. The reason is that the allowance price increases in the demand shock θ, because the firms must reduce their emission intensity in order to increase production in response to the positive demand shock under emissions trading. Hence, the higher product price following a positive shock to demand is partly offset by a higher price on emissions under ETS. This contrasts with an emission tax, where the firms pay a fixed price on emissions regardless of the aggregate emissions level. Last, the shock to abatement cost (φ) has a negative impact on production under emissions trading, because abatement becomes more costly. Aggregate production is independent of the shocks to abatement costs under an emissions tax, because the marginal cost of emissions is equal to the fixed emissions tax which does not depend on φ. The economic intuition regarding the result on the product price in Lemma 2 is very similar.

Remember that the variances of the aggregate shocks, featured in Lemma 2, are functions of the variances of the individual shocks and the correlation coefficients (e.g., $n\left(1+(n-1)\rho_\eta\right)\sigma_{\eta_i}^2 \equiv \sigma_\eta^2$). It follows that the variance in aggregate production and the product price tends to be larger under an emissions tax (as compared with ETS) if the shocks to the firms' production costs (η_i) and consumer utility (θ_j) have large variances and are strongly positively correlated (ρ_η and ρ_θ). Conversely, the variance in product prices tends to be lower under TAX if the shocks to firms abatement cost (φ_i) have large variance and strong positive correlation (ρ_φ).[7]

Lemma 2 relates to Mendelsohn (1984) and Krysiak (2008), which both shows that quantity-based instruments have an advantage over price-based regulation, because the latter induces excessive variation in output. The result in Lemma 2 differs mainly because the theory framework in the present paper features both a product market and a market for emissions allowances.

We have the following result:

Proposition 1. *The volatility of aggregate production increases (decreases) in the price elasticity of the supply of emission allowances (v) if and only if $\sigma_\eta^2 + \sigma_\theta^2$ is sufficiently large (small) as compared with σ_φ^2.*

Proof. *The proposition follows from Lemma 1 and Lemma 2.*

Proposition 1 highlights the role that a price elastic supply of emission allowances may have on the product market. Specifically, more price elastic supply of emission allowances involves increased volatility in aggregate production if the shocks to consumer demand and production costs together are sufficiently large as compared with the shock to abatement costs. This is

[7] Hintermann (2010) examines the extent to which variation in the EU ETS allowance price can be explained by marginal abatement costs during the first phase of the EU ETS. He finds that the most important allowance price determinants after the allowance price crash in April 2006 was fuel prices, summer temperatures, and precipitation.

relevant, e.g., if allowance price stabilizing measures are introduced to reduce allowance price volatility, and thereby create a less risky environment for investment in abatement technologies.[8] The reason is that efforts to reduce volatility in the emission allowance market may come at the cost of a more risky investment environment in the product market. For example, Proposition 1 implies that the volatility in the price on electricity in the area covered by the EU ETS will be affected if emission allowance stabilizing measures such as the Market Stability Reserve (MSR) are introduced or expanded. Whether the volatility increases or decreases depends on the characteristics of the uncertainty (as described by Proposition 1).

The volatility in aggregate emissions unambiguously increases in the parameter determining the price elasticity of permit supply v; see equation (9) and the reduced form solutions for aggregate emissions in Appendix A. Conversely, the volatility in the price on emissions unambiguously decreases in v. Lemma 2 and and Proposition 1 are valid for any allowance price target $\widetilde{\tau}$ (i.e., they do not require the assumption $\widetilde{\tau} = E(\tau^{ets})$).

So far, the focus has been on aggregate production and emissions. What matters for the firms' investment levels, however, are the first and second order moments of their individual production and abatement levels (see below). In order to establish a reasonable basis for comparison, suppose that $\widetilde{\tau}$ in equation (3) are set such that the expected price on emissions are equal to the equilibrium price under ETS ($v = 0$) across the regulatory schemes. I show in Appendix A that the firms individual production and abatement is then given by:

$$\begin{pmatrix} q_i^{ets} & a_i^{ets} \\ q_i^{hyb} & a_i^{hyb} \\ q_i^{tax} & a_i^{tax} \end{pmatrix} = \begin{pmatrix} \overline{q}_i + \Psi_{q_i}^{ets}(\eta, \varphi, \theta) & \overline{a}_i + \Psi_{a_i}^{ets}(\eta, \varphi, \theta) \\ \overline{q}_i + \Psi_{q_i}^{hyb}(\eta, \varphi, \theta) & \overline{a}_i + \Psi_{a_i}^{hyb}(\eta, \varphi, \theta) \\ \overline{q}_i + \Psi_{a_i}^{tax}(\eta, \theta) & \overline{a}_i + \Psi_{a_i}^{tax}(\varphi) \end{pmatrix}. \tag{10}$$

Here, $\overline{q}_i$ and $\overline{a}_i$ are each firm's expected production and abatement, which is equal across firms. The Ψ's represent the stochastic elements, which differ across firms (see Appendix A for the exact expressions). Remember that emissions are a linear combination of production and abatement ($\varepsilon_i = kq_i - a_i$). Whereas the expected terms are equal across the schemes, the stochastic elements differ. In particular, we have:

$$\begin{pmatrix} \Psi_{q_i}^{ets} & \Psi_{q_i}^{tax} \\ \Psi_{a_i}^{ets} & \Psi_{a_i}^{tax} \end{pmatrix} = \begin{pmatrix} \frac{\left(dn+k^2m\gamma\right)(\eta-n\eta_i)-mn\beta\eta_i-km\beta\varphi+n\theta\beta}{m\gamma k^2n\beta+dn^2\beta+mn\beta^2} & \frac{d\eta+\theta\beta}{\beta(m\beta+dn)} - \frac{1}{\beta}\eta_i \\ \frac{-km\gamma\eta+(m\beta+dn)(\varphi-n\varphi_i)-k^2mn\gamma\varphi_i+kn\theta\gamma}{mk^2n\gamma^2+dn^2\gamma+m\beta n\gamma} & -\frac{1}{\gamma}\varphi_i \end{pmatrix}, \tag{11}$$

in equation (10), with the stochastic elements under HYB being somewhere in between those of

[8] This is arguably a motivating factor for the introduction of the EU ETS Market Stability Reserve. See also Grüll and Taschini (2011).

ETS and TAX (depending on v), cf. Lemma 1.[9] Equations (10) and (11) highlight the different environments with respect to risk imposed by the different regulatory regimes. Specifically, firms are not exposed to the variance in other firms' abatement costs under TAX, because the allowance price is fixed. Moreover, firms' production under TAX is independent of the uncertainty in the firms' own abatement costs, because the marginal cost of emissions is exogenously given by the emissions tax. Last, the firms' abatement levels (but not emissions) is independent of the shocks to production costs and consumer utility under TAX. Does this mean that firms are less exposed to risk, interpreted as higher variances in production and abatement levels, under TAX? We have the following result on the relative variances in individual firms production across the regulatory schemes:

Lemma 3. *Suppose* $\widetilde{\tau} = E(\tau^{ets})$ and *technology* $\mathbf{x}$ *is fixed and equal for all* v*. Then we have the following:*

(i) *If* $\sigma^2_{\varphi_i} = \sigma^2_{\theta_i} = 0$ *and* $\sigma^2_{\eta_i} > 0$ *we have* $var(q_i^{ets}) < (=) var(q_i^{hyb}) < (=) var(q_i^{tax})$*, given* $\rho_\eta > (=) - 1/(n-1)$.

(ii) *If* $\sigma^2_{\eta_i} = \sigma^2_{\theta_i} = 0$ *and* $\sigma^2_{\varphi_i} > 0$ *we have* $var(q_i^{tax}) < (=) var(q_i^{hyb}) < (=) var(q_i^{ets})$*, given* $\rho_\varphi > (=) - 1/(n-1)$ *and finite* n *and* m.

(iii) *If* $\sigma^2_{\eta_i} = \sigma^2_{\varphi_i} = 0$ *and* $\sigma^2_{\theta_i} > 0$ *we have* $var(q_i^{ets}) < (=) var(q_i^{hyb}) < (=) var(q_i^{tax})$*, given* $\rho_\theta > (=) - 1/(n-1)$ *and finite* n.

Proof. *See Appendix A.*

Lemma 3 corresponds to the conclusion from Proposition 1, but this time for individual firms. Lemma 3 implies that the variation in each individual firm's production can increase in the price elasticity of emission allowances (v in equation (3)) if the shocks to consumer demand and production costs together are sufficiently large, as compared with the shock to abatement costs. Conversely, the variation in production decreases in v if the shocks to abatement costs are the dominant factor. This in turn has implications for the firms' risk environment and their choice of production technology in Stage 2 (see Section 2.2 below). If several shocks are present at the same time, the relative size of the variances ($\sigma^2_{\varphi_i}$, $\sigma^2_{\theta_i}$ and $\sigma^2_{\eta_i}$) and the correlation coefficients (ρ_η, ρ_φ and ρ_θ) will together be decisive.

[9] The expressions are less tractable under HYB, in particular when calculating, e.g., variances based on equation (11).

2.2 Stage 2: The firms' investment decisions

In Stage 2, any firm $i \in I$ maximizes expected profits with respect to cost structure, as determined by the technology parameters $\mathbf{x} = \{\alpha, \beta, \delta, \gamma, \widetilde{q}, \widetilde{a}\}$:

$$\max_{\mathbf{x}} \left(E\left(\pi_i\right) - \kappa\left(\mathbf{x}\right)\right), \tag{12}$$

with π_i given by equation (2). The maximand in equation (12) is a sum of linear and strictly concave functions, implying that the maximization problem is strictly concave and hence has a unique global maximum. The maximization in equation (12) is done under uncertainty, which contrasts with the firms' optimization problem (2) in Stage 3. The interior solution to the maximization problem (12) is characterized by the following first-order conditions (where I used the envelope theorem in the derivation):

$$\begin{pmatrix} -\kappa^*_\alpha & -\kappa^*_\delta \\ -\kappa^*_\beta & -\kappa^*_\gamma \\ -\kappa^*_{\widetilde{q}_i} & -\kappa^*_{\widetilde{a}_i} \end{pmatrix} = \begin{pmatrix} E\left(q_i\right) & E\left(a_i\right) \\ \frac{1}{2}E\left(\left(q_i - \widetilde{q}_i\right)^2\right) & \frac{1}{2}E\left(\left(a_i - \widetilde{a}\right)^2\right) \\ \beta_i E\left(q_i - \widetilde{q}_i\right) & \gamma_i E\left(a_i - \widetilde{a}_i\right) \end{pmatrix}, \tag{13}$$

where an asterisk '$*$' indicates that the level solves the firms technology optimization problem (12). Note that the expectations $E\left(q_i\right)$ and $E\left(a_i\right)$ in Stage 2 are equal across firms, even tough realized abatement and production in Stage 3 differ between firms. This is why the firms chose equal technology in Stage 2 (across firms, not regulatory schemes).

We see that investment that reduces abatement and production unit costs (α and δ) increases in expected production and abatement. For example, the more you expect to produce, the more you are willing to invest to decrease the unit cost of production. Further, the production flexibility parameter β is set such that it becomes less expensive to deviate from minimum efficient scale (MES) production, as determined by $\widetilde{q}_i$, if the variances in production increases. The logic is similar for the abatement flexibility parameter γ, which increases in the variance of the firm's abatement.[10] Last, the MES parameters ($\widetilde{q}_i$ and $\widetilde{a}_i$) are set such that MES are close to (but below) expected production and abatement. The difference between MES and expected production (abatement) increases in investment cost, and decreases in the cost of producing (abating) at a another production level determined by β (γ).

We know from equation (9) and Proposition 1 that the second order moments of production and abatement differs across the regulatory regimes, even if regulation is rigged such that $E(\chi^{ets}) = E(\chi^{hyb}) = E(\chi^{tax})$, i.e. by setting $\widetilde{\tau} = E(\tau^{ets})$ (cf., Lemma 1). It follows that the regulatory regimes induces different types of technology.

[10]We have $\frac{1}{2}E\left(\left(q_i - \widetilde{q}_i\right)^2\right) = \frac{1}{2}\left(var(q_i - \widetilde{q}_i) + \left(E\left(q_i - \widetilde{q}_i\right)\right)^2\right)$ in equation (13), and similarly for abatement.

Suppose the firms have adapted to a standard ETS ($v = 0$) with the associated optimal technology. Then introducing elastic permit supply ($v > 0$) entails investment costs, because the firms must reconfigure their technology to the risk environment induced by the new type of regulation. This cost, associated with changing regulatory regimes, may be a part of the discussion when considering regulatory changes.

2.3 Stages 1 and 2: Welfare

I assume that the social planner first determines the regulatory parameters v, $\widetilde{\tau}$ and S in equation (3) in Stage 1. These parameters may correspond to optimal policy, but the results in this paper are valid for any parameter values unless otherwise stated.

The optimal policy in Stage 3 under ETS and TAX would be the well-known condition that the emission cap or allowance price equalizes expected marginal environmental damages with expected marginal abatement costs. Note that the product market does not need any interventions from the regulator except for the emissions regulation (cf., perfect competition and the first theorem of welfare). The optimal v in equation (3) under HYB, i.e. the v that maximizes expected welfare given the firms' investment decisions, may be less obvious, however.

Suppose a beneficial social planner maximizes expected welfare with respect to v in equation (3), subject to the actions of the firms and consumers in stages 2 and 3. The social planner's maximization problem is given by:

$$W_v = \max_v E\left[\sum_{j\in J} u_j\left(q_j, \theta_j\right) - \sum_{i\in I}\big(c(q_i, a_i; \mathbf{x}_i, \eta_i, \varphi_{i,}) + \kappa\left(\mathbf{x}\right)\big) - g\left(\varepsilon\right)\right],$$

where $g\left(\varepsilon\right) = \sum_{j\in J} g_j\left(\varepsilon\right)$ and utility, $u_j\left(q_j, \theta_j\right)$, and production cost, $c(q, a; \mathbf{x}_i, \eta_i, \varphi_i)$, are given by equations (4) and (1), respectively. It can be shown that the socially optimal interior solution for v in equation (3) satisfies (see the Appendix):[11]

$$E\left[g_\varepsilon \frac{d\varepsilon}{dv} + \tau \frac{da}{dv}\right] = 0. \tag{14}$$

We first observe that aggregate emissions and aggregate abatement moves in opposite directions inside the parenthesis in equation (14) (see the derivation of equation 14 in the Appendix). Suppose emissions increase in v (and abatement decreases). Then the first order condition (14) states that v is calibrated such that the expected increase in environmental damage equals the expected savings from less abatement. If this where not true, expected welfare could be increased by changing v. For example, if marginal environmental damages decline more than

[11] Attempts to derive reduced form solutions for v leads to large analytical expressions that are hard to interpret.

the associated increase in (aggregate) marginal abatement cost following a decrease in v, total welfare could be increased by reducing v. Whereas the requirement that the marginal change in expected welfare followed by a marginal increase in v is zero at optimum may not be that surprising, we observe that investment costs do not enter equation (14) directly. Investment costs are indirectly present as they determine the marginal changes in emissions and abatement inside the parenthesis in equation (14), however. We last note that a corner solution where HYB collapses to ETS and $v = 0$ is also possible, and that HYB converges towards a tax as v becomes very large.

To examine socially optimal technology investment in Stage 2, suppose a beneficial social planner maximizes expected welfare with respect to the technology parameters $\mathbf{x} = \{\alpha, \beta, \delta, \gamma, \widetilde{q}, \widetilde{a}\}$, subject to the competitive equilibrium in Stage 3. The social planner's maximization problem is given by:

$$W_{\mathbf{x}} = \max_{\mathbf{x}} E\left[\sum_{j\in J} u_j\left(q_j,\theta_j\right) - \sum_{i\in I}\left(c(q_i,a_i;\mathbf{x}_i,\eta_i,\varphi_{i,}) + \kappa\left(\mathbf{x}\right)\right) - g\left(\varepsilon\right)\right],$$

where utility, $u_j\left(q_j,\theta_j\right)$, and production cost, $c(q,a;\mathbf{x}_i,\eta_i,\varphi_i)$, are given by equations (4) and (1), respectively. Further, assume that S and/or $\widetilde{\tau}$ in equation (3) is set such that $E\left(g_\varepsilon\right) = E\left(\tau\right)$ and assume quadratic environmental damage $g\left(\varepsilon\right) = g_1\varepsilon + g_2\varepsilon/2$. Then we have the following (see Appendix A):

$$\begin{pmatrix} -\kappa_\alpha^{sp} & -\kappa_\delta^{sp} \\ -\kappa_\beta^{sp} & -\kappa_\gamma^{sp} \\ -\kappa_{\widetilde{q}_i}^{sp} & -\kappa_{\widetilde{a}_i}^{sp} \end{pmatrix} = \begin{pmatrix} -\kappa_\alpha^{*} & -\kappa_\delta^{*} \\ -\kappa_\beta^{*} - Cov\left(\frac{g_\varepsilon-\tau}{n},\frac{d\varepsilon}{d\beta_i}\right) & -\kappa_\gamma^{*} - Cov\left(\frac{g_\varepsilon-\tau}{n},\frac{d\varepsilon}{d\gamma_i}\right) \\ -\kappa_{\widetilde{q}_i}^{*} & -\kappa_{\widetilde{a}_i}^{*} \end{pmatrix}, \quad (15)$$

where superscript 'sp' denotes the socially optimal technology investment levels and the variables with asterisk are given by equation (13). The above assumption that $E\left(g_\varepsilon\right) = E\left(\tau\right)$ states that the regulator chooses the policy parameters in Stage 1 such that the expected price on emissions equals expected marginal environmental damage in Stage 3 (optimal policy). If this assumption is violated, the socially optimal technology investment levels differ from those of the firms for all parameters, not only β and γ. The reason is that the social planner then uses the technology to compensate for the suboptimal regulatory policy in Stage 3. The difference between the socially optimal technology and the firms' investment is given by the covariances $Cov\left(\frac{g_\varepsilon-\tau}{n},\frac{d\varepsilon}{d\beta_i}\right)$ and $Cov\left(\frac{g_\varepsilon-\tau}{n},\frac{d\varepsilon}{d\gamma_i}\right)$ in equation (15). Interestingly, these terms are zero in the case of (i) standard ETS ($v = 0$), because the fixed and binding emissions cap S implies that $d\varepsilon/d\beta = d\varepsilon/d\gamma = 0$, and ($ii$) linear environmental damage (given the assumption that $E\left(g_\varepsilon = \tau\right)$).

We have the following result:

Proposition 2. *Let environmental damage be quadratic and strictly convex in aggregate emissions. Then, the firms technology choice is socially optimal in the presence of uncertainty if and only if the permit supply is perfectly inelastic ($v = 0$).*

Proof. *The proposition follows from equation (15).*

The term 'uncertainty' in Proposition 2 is interpreted such that at least one of the stochastic shocks η, φ and θ is present, and that not all shocks present are correlated such that they cancel each other out. Proposition 2 generalizes the results of Krysiak (2008) and Storrøsten (2014; 2015) to hybrid regulation, stochastic shocks to production costs (as opposed to abatement cost shocks only) and a more general cost structure.[12] Proposition 2 is valid also if $E(g_\varepsilon) \neq E(\tau)$ (see Appendix A).

Proposition 2 entails that endogenous technology choice provides a comparative advantage for ETS over HYB (for any given $v > 0$), and HYB over TAX (for any finite v). The strength of this advantage increases in the absolute value of the magnitude of the variances of the shocks to the demand (θ) and supply (η and φ) sides in the economy, as well as the correlation between these shocks (the ρ's). It also increases in the convexity of environmental damages.[13] One implication of this is that allowance stabilizing measures such as the Market Stability Reserve (MSR), if implemented in a standard ETS ($v = 0$), introduces a negative technology externality, and that the strength of this externality is determined by the characteristics of the uncertainty and the convexity of the environmental damage function. In this context, it is important to distinguish between the optimization of welfare and the optimization of technology. Specifically, it may increase expected welfare to introduce a price-elastic supply of emission allowances (e.g., moving from ETS to HYB), even if this involves suboptimal investments in technology. The reason is that the benefits from elastic supply of emission allowances may very well outweigh the loss from the negative technology externality.

Proposition 2 suggests that technology policies can improve welfare if and only if the regulatory scheme features endogenous aggregate emissions.[14] Assume TAX regulation, with the tax set equal to the marginal environmental damage from expected emissions, $E(g_\varepsilon) = \tau^{tax}$. Further, let environmental damage be strictly convex and given by $g(\varepsilon) = g_1\varepsilon + g_2\varepsilon/2$ with $g_1 > 0$ and $g_2 > 0$. We then have the following result on technology investment policies under TAX:

[12] The current paper has six endogenous technology variables, whereas Storrøsten (2014; 2015) and Krysiak (2008) have two endogenous technology parameters.

[13] Note that these issues are reflected in the expressions for the optimal subsidies in Proposition 3 below.

[14] Remember that other potential reasons for technology subsidies, like, e.g., technology spillovers or learning by doing, are not present in the analysis.

Proposition 3. Assume $E(g_\varepsilon) = \tau^{tax}$ and $g(\varepsilon) = g_1\varepsilon + g_2\varepsilon/2$. Then, the optimal taxes on investment in β and γ under TAX are given by:

$$\begin{aligned} t_\beta^{tax} &= g_2 \frac{k^2 m}{(m\beta + dn)^3 n} \left(\left(n^2 \left(n + n(n-1)\rho_\theta \right) \sigma_{\theta_i}^2 + m^2 \left(n + n(n-1)\rho_\eta \right) \sigma_{\eta_i}^2 \right) \right) \geq 0, \\ t_\gamma^{tax} &= \frac{g_2}{\gamma^2} \frac{dn}{d\gamma n^2 + m\beta\gamma n} \left(n + n(n-1)\rho_\varphi \right) \sigma_{\varphi_i}^2 \geq 0, \end{aligned}$$

respectively. It is not optimal to tax or subsidize investment in the other technology parameters (i.e., we have $t_\alpha^{tax} = t_\delta^{tax} = t_{\widetilde{q}}^{tax} = t_{\widetilde{a}}^{tax} = 0$).

Proof. *See Appendix A.*

Proposition 3 gives the taxes that induce the firms to invest in the socially optimal technology, given optimal policy and the assumed quadratic environmental damage function.

A key message from Proposition 3 is that, in the presence of uncertainty, endogenous aggregate emissions and strictly convex environmental damage, it is optimal to tax investment in technology that increases the variance in emissions. The reason is that fluctuations in emissions creates a negative externality that is not internalized by the firms. More precisely, the cost of emissions is strictly convex from the social planner's perspective (by assumption). Hence, the expected damage from emissions is larger than the damage from expected emissions from a welfare perspective, cf. Jensen's inequality and convex environmental damage. The firms, on the other hand, face a linear cost of emissions, which is simply given by the fixed tax multiplied with their emissions level. Consequently, the firms do not consider the increased environmental damage that follows from the fluctuations of emissions around their mean. The fact that these fluctuations are influenced by the firms' technology decisions is what constitutes the negative externality that motivates the taxes on investment in the flexibility technology parameters β and γ in Proposition 3. Note that, in the case of t_γ^{tax}, Proposition 3 involves taxation of investment in abatement technology.

We observe that the optimal tax to investment in γ^{tax} is zero if $\sigma_{\varphi_i}^2 = 0$, if $\rho_\varphi = -1/(n-1)$, or if $\rho_\varphi = 0$ and $n \to \infty$. The explanation is that the part of the variance in emissions that is influenced by γ^{tax} collapses to zero in these cases. Hence, there is no need to regulate investment in γ^{tax} (the rationale for the investment tax is to reduce variance in emissions and thereby environmental damage). By the same reasoning, the optimal tax to investment in β^{tax} is zero if the uncertainty parameters are such that the uncertainty that is affected by β^{tax} disappears. We also observe that Proposition 3 would prescribe a subsidy to investment on β and γ in the case of concave environmental damage (in which case we have $g_2 < 0$ in Proposition 3). Whereas this may not be very realistic in the case of environmental damage, it may be relevant for some other public goods.

Proposition 3 indicates that it is also optimal to regulate investment in β and γ under HYB. That is, even though the emission price is under HYB is endogenous, marginal environmental damage is endogenous and may differ from the price on emissions. Derivations of expressions for the optimal subsidies do not lead to interpretable analytical results, however (the expressions become too large). We nevertheless observe that the optimal taxes under HYB satisfy $t_{\beta}^{hyb} = t_{\gamma}^{hyb} = 0$ if $v = 0$, $\lim_{v\to\infty}\left(t_{\beta}^{hyb}\right) = t_{\beta}^{tax}$, and $\lim_{v\to\infty}\left(t_{\gamma}^{hyb}\right) = t_{\gamma}^{tax}$ (cf. Lemma 1).

Last, it is worth emphasizing that the optimal tax scheme in Proposition 3 depends strongly on the functional form of the cost function (1). For other technology structures, the expressions will change, and may even change sign from a tax to a subsidy. As such, the key lessons from Proposition 3 is that (*i*) implementation of the socially optimal cost structure may require technology policies (e.g., taxes, subsidies or technology standards) even without the presence of issues like positive technology externalities or market power. And, (*ii*), that these technology policies should aim at reducing variation in aggregate emissions.

2.4 Interpretation of the results on investment in the context of Weitzman (1974)

The theory model in this paper is quite different from that of Weitzman (1974), but the results are nevertheless closely related to his analysis. They are also perhaps easier to understand and put in perspective if placed into that well-known framework.

As in Weitzman (1974), the present analysis features a model with polluters (the firms) with marginal benefits (or savings) from emissions that are unknown to the regulator. The benefits are stochastic but known to the polluting firm when it chooses emissions (in Stage 3). We have a non-stochastic marginal environmental damage function that is known to the regulator.[15] Weitzman (1974) considers the following research problem: Does price or quantity-based regulation induce the highest expected welfare?

Weitzman's problem is illustrated in Figure 1. Here marginal damages (MD) are increasing in emissions, whereas expected (E(MB)) and realized marginal benefits (MB) from emissions are decreasing in emissions. Note first that without uncertainty we have MB = E(MB) and the optimal allocation is given by point C in Figure 1 (i.e., no shift to MB).[16] At C the sum of environmental damages (the area BCKA) and emission reductions (KCI) is minimized. This can be achieved with either the TAX or the emission CAP graphed in Figure 1, so price and quantity-based regulation performs equally well. But this changes with uncertainty. In Figure

[15] Note that the uncertainty in benefits does not appear in the well-known Weitzman (1974) criterion comparing price versus quantity-based regulation. As pointed out by Weitzman (1974), the reason is that the expected benefit function does not depend on the variance of marginal benefits so long as costs and benefits are independently distributed.

[16] This corresponds to the case with $\sigma_{\eta}^2 = \sigma_{\theta}^2 = \sigma_{\varphi}^2 = 0$ in the framework of the present paper.

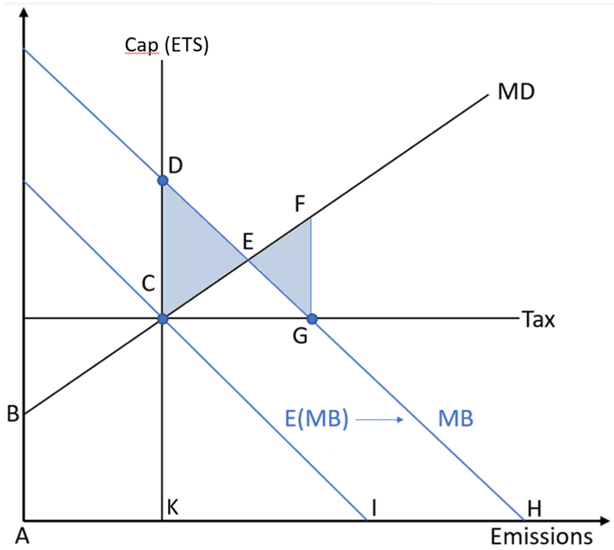

Figure 1: Illustration of prices versus quantities in Weitzman (1974).

1 I consider a positive shock to the marginal benefits from emissions, but the case is analogous with a negative shock. The optimal allocation is now given by E, but price and quantity-based regulation based on expected marginal benefits induces the suboptimal outcomes G and D, respectively.

The optimal allocation E is associated with a total cost (i.e., the cost of damages plus emission reductions) given by the area ABEH. In comparison price and quantity-based regulation induces total costs equal to the areas BFGHA and BCDHA, respectively. The welfare loss associated with quantity-based regulation is given by the shaded area CDE, whereas the loss associated with price-based regulation is given by the area EFG (both as compared with the optimal allocation E). We see that price and quantity-based regulation are no longer equal in the presence uncertainty, and that the relative performance of the instruments depends on the slopes of the marginal damage and emission reduction functions. Specifically, Weitzman (1974) finds that quantity-based regulation (ETS) outperforms price-based regulation (TAX) if marginal damages are more steeply sloped than marginal benefits from emissions. Conversely, price-based regulation induces the highest expected welfare if marginal benefits from emissions are more steeply sloped than marginal damages. Figure 1 is drawn such that price-based regulation induces highest expected welfare.

One important difference between the present paper and the analysis in Weitzman (1974) is that I examine technology investment. More precisely, the present paper features three stages: (1) choice of regulatory regime, (2) technology investment, and (3) production and emissions, with the stochastic variables determined in between Stage 2 and 3. The Weitzman (1974) analysis does not feature the investment stage (2). In context of Figure 1, the marginal benefit (MB) function is endogenous in the present paper, but exogenous in Weitzman (1974). This clearly implies that the above-mentioned criterion in Weitzman (1974) on prices vs. quantities is not directly applicable.

Whereas the marginal damage function graphed in Figure 1 is similar to that of Weitzman (1974) in the present paper, the interpretation of marginal benefits from emissions is slightly more involved. The main reason is that this analysis includes a product market. That is, whereas the benefits from emissions mirrors lower abatement costs in the present analysis, they also depend on the product market. For example, everything else equal, larger demand or lower supply costs for the good increases the marginal benefits from emissions. The marginal benefits from emissions also increase in marginal abatement costs. With this in mind, the marginal benefits in Figure 1 may be interpreted as the benefits from emissions (from a firm aggregate) in the present analysis. Importantly, not only the slope and intercept of the MB function is endogenous, but also the impact of the various shocks featured in the model (indicated by the length of the arrow in Figure 1).

We know from equations (13) and (15) that the parameters determining the firms' unit production and abatement costs at minimum efficient scale (MES, α, δ), and the MES levels ($\widetilde{q}$, $\widetilde{a}$) are socially optimal regardless of regulatory regime. The parameters that determine the firms' flexibility in production and abatement (β, γ) are socially suboptimal, unless the supply of emission allowances is exogenous (ETS), however. In terms of Figure 1, investment in a lower β and γ tends to increase the size of the shift in MB caused by uncertainty (the length of the arrow).

The firms benefit from a flexible production technology that allows higher production when production costs are low ($\sigma_\eta^2 < 0$) or demand high ($\sigma_\theta^2 > 0$), and vice versa. Similarly, a flexible abatement technology allows for less abatement when abatement costs turn out to be high ($\sigma_\varphi^2 > 0$). The firms pay for this flexibility when investing in β and γ in Stage 2 (cf., equation 13). The associated increase in emission fluctuations is not a concern for the firms under TAX. In contrast, the regulator faces a convex damage function. Lower fluctuations in emissions hence reduces expected environmental damage. The regulator therefore balances the benefits from flexible technology against the cost of higher variance in aggregate emissions. The firms overinvestment in flexibility increases the welfare loss indicated by the area EFG in Figure 1 under TAX.

Why do the firms' investment decisions perfectly internalize the welfare loss under ETS, but not under TAX? Examining the ETS welfare loss area DCE, we see that it takes the form of too high abatement costs. This is paid for and, hence, internalized by the firms. In contrast, the welfare loss under TAX, EFG, is caused by too high environmental damage. This is shouldered by the consumers and thus not internalized by the firms. Therefore, the firms' investment is suboptimal in the case of a tax, and socially optimal under ETS. This issue, i.e., whether the welfare loss is burdened on the firms or the consumers, is not central in Weitzman (1974). The reason is that it only has a distributional impact and does not affect overall welfare (in the Weitzman (1974) model with exogenous technology). Yet, it is essential for the results on technology investment and active technology policies in the present paper.

We last note that expected welfare may well be higher under ETS than under TAX even if the choice of technology under TAX is suboptimal (from a welfare perspective). An analogue to the well-known Weitzman (1974) criterion on prices vs. quantities would have to take the endogeneity of the marginal cost function into account. Whereas the present paper does not derive such a criterion, the optimal policy is given by the hybrid policy with v as in equation (14) (see also Storrøsten, 2015).[17]

3 Concluding remarks

Implementation and development of new technologies are essential for tackling environmental challenges such as climate change, acid rain and depletion of the ozone layer. In this paper I examined the investment incentives of market-based regulation, with focus on the technology characteristics the different regulatory schemes tend to drive forward. The paper also investigated the rationale for active technology policies. I showed that, in the presence of uncertainty, endogenous aggregate emissions and strictly convex environmental damage, it is optimal to tax investment in technologies that induce large variance in emissions. I also examined how elastic supply of emission allowances influences the volatility of product prices and derived the optimal hybrid policy with price elastic supply of emission allowances (and endogenous technology). I found that price elastic supply of emission allowances involves increased volatility in the product market if the shocks to consumer demand and production costs are sufficiently large, as compared with the shocks that hit the abatement part of the firms cost function.

The results are derived under rather strict assumptions on the functional forms. In particular, the cost function is separable in production and abatement, implying that abatement is modelled as end of pipe technology. Whereas the mechanisms examined would still be present with a more general cost function, the results would also be contingent on the cross derivatives between abatement and production. The results in the present paper would remain valid, unless the cross derivatives where significantly large and with the opposite sign as compared with the direct effect, however.

[17] The hybrid scheme (HYB) is somewhere in between these two regulatory regimes. Investment under HYB is socially suboptimal, but closer to the optimal investment than under TAX.

HYB cannot perform worse than TAX or ETS under optimal policy, since HYB is more flexible and can replicate both if appropriate. Nevertheless, the results in the present paper indicate that regulatory schemes with endogenous aggregate emissions have a disadvantage as compared with regulation with fixed aggregate emissions in the presence of endogenous technology choice and convex environmental damage. Specifically, introduction of permit price stabilizing measures in an emission trading system will come at the cost of suboptimal technology investments, and may also cause increased fluctuations in product prices (which in itself is not necessarily negative). Such considerations may be particularly relevant if the regulator has imperfect information and optimal policy, including the taxes on investment in Proposition 3, is difficult to implement.

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Appendix A: proofs and derivations

Solution to stage 3: The first-order conditions to the maximization problem (5) yields the following solution for production q_i and abatement a_i:

$$\begin{aligned} q_i &= \frac{1}{\beta}\left(p - \alpha - \eta_i - k\tau + \beta\widetilde{q}_i\right), \\ a_i &= \frac{1}{\gamma}\left(\tau - \delta_i - \varphi_i + \gamma\widetilde{a}_i\right), \end{aligned}$$

which together with $\varepsilon_i = kq_i - a_i$ yields equations (6) and (7). The first-order conditions to the consumer maximization problem gives:

$$q_j = \frac{1}{d}\left(b - p + \theta_j\right),$$

which yields equation (8).

The competitive equilibrium solves the first order conditions (5), (6) and (7) subject to the market equilibrium conditions (3) and (8). Define $K^{ets} = 1/\left(\left(m\beta + dn + k^2 m\gamma\right)\right)$. Under ETS ($v = 0$ in equation 3), the reduced form solutions to prices and aggregate production and emissions in stage 3 are given by:

$$\begin{aligned} p^{ets} &= K^{ets}\left(bm\beta + dn\alpha - Sdk\gamma - \widetilde{q}_i dn\beta + dkn\delta + bk^2 m\gamma - \widetilde{a}_i dkn\gamma\right) + K^{ets}\left(d\eta + dk\varphi + \theta\left(\beta + k^2\gamma\right)\right) \\ &= \overline{p} + \Psi_p^{ets}, \\ q^{ets} &= K^{ets}\left(-n\alpha + bn + Sk\gamma + \widetilde{q}_i n\beta - kn\delta + \widetilde{a}_i kn\gamma\right)m + K^{ets}\left(-km\varphi - m\eta + n\theta\right), \\ \tau^{ets} &= \frac{K^{ets}}{n}\left(dn^2\delta - Sdn\gamma - Sm\beta\gamma + mn\beta\delta - \widetilde{a}_i dn^2\gamma + bkmn\gamma - \widetilde{a}_i mn\beta\gamma - kmn\alpha\gamma + \widetilde{q}_i kmn\beta\gamma\right) \\ &\quad + \frac{K^{ets}}{n}\left(\varphi\left(m\beta + dn\right) - km\gamma\eta + kn\theta\gamma\right) \\ &= \overline{\tau} + \Psi_\tau^{ets}, \\ \varepsilon^{ets} &= S, \end{aligned}$$

where $\overline{p}$ and $\overline{\tau}$ refer to the expected values and the stochastic elements are represented by the Ψ's. Define $K^{hyb} = 1/\left(dn^2 + mn\beta + dnv\gamma + mv\beta\gamma + k^2 mn\gamma\right)$. Then, the solutions under HYB are given by:

$$\begin{aligned}
p &= K^{hyb}\left(n\left(bm\beta+dn\alpha-\widetilde{q}_i dn\beta+dkn\delta\right)+\gamma\left(bmv\beta+dnv\alpha-\widetilde{a}_i dkn^2+bk^2mn-Sdkn\right)\right)\\
&\quad +K^{hyb}\left(-\widetilde{q}_i dnv\beta+dknv\widetilde{\tau}\right)\\
&\quad +K^{hyb}\left(dkn\varphi+d\eta\left(n+v\gamma\right)+\theta\left(n\beta+v\beta\gamma+k^2n\gamma\right)\right)\\
q &= K^{hyb}\left(bmn^2-mn^2\alpha+\widetilde{q}_i mn^2\beta-kmn^2\delta+Skmn\gamma+bmnv\gamma-mnv\alpha\gamma+\widetilde{a}_i kmn^2\gamma\right)\\
&\quad +K^{hyb}\left(\widetilde{q}_i mnv\beta\gamma-kmnv\widetilde{\tau}\gamma\right)\\
&\quad +K^{hyb}\left(-kmn\varphi-m\eta\left(n+v\gamma\right)+n\theta\left(n+v\gamma\right)\right)\\
\tau &= K^{hyb}\left(dn^2\delta-Sdn\gamma-Sm\beta\gamma+mn\beta\delta-dn^2\gamma+bkmn\gamma-\widetilde{a}_i mn\beta\gamma-kmn\alpha\gamma+dnv\widetilde{\tau}\gamma\right)\\
&\quad +K^{hyb}\left(mv\beta\widetilde{\tau}\gamma+\widetilde{q}_i kmn\beta\gamma\right)\\
&\quad +K^{hyb}\left(dn\varphi+m\beta\varphi+kn\theta\gamma-km\gamma\eta\right)\\
\varepsilon &= K^{hyb}\left(Sdn^2+Smn\beta-dn^2v\widetilde{\tau}+dn^2v\delta-\widetilde{\tau}mnv\beta+mnv\beta\delta-\widetilde{a}_i dnv\gamma+Sk^2mn\gamma+bkmnv\gamma\right)\\
&\quad +K^{hyb}\left(-\widetilde{a}_i mnv\beta\gamma-kmnv\alpha\gamma-k^2mnv\widetilde{\tau}\gamma+\widetilde{q}_i kmnv\beta\gamma\right)\\
&\quad +K^{hyb}\left(dnv\varphi+mv\beta\varphi+knv\theta\gamma-kmv\gamma\eta\right)
\end{aligned}$$

Define $K^{tax}=1/\left(m\beta+dn\right)$. Then the solutions under TAX are given by (note that $v\to\infty$ in equation 3 requires $\tau=\widetilde{\tau}$):

$$\begin{aligned}
p^{tax} &= K^{tax}\left(bm\beta+dn\alpha-\widetilde{q}_i dn\beta+\widetilde{\tau}dkn\right)+K^{tax}\left(d\eta+\theta\beta\right)\\
q^{tax} &= K^{tax}\left(mn\left(b-\alpha+\widetilde{q}_i\beta-\widetilde{\tau}k\right)\right)+K^{tax}\left(n\theta-m\eta\right)\\
\tau^{tax} &= \widetilde{\tau}\\
\varepsilon^{tax} &= K^{tax}\left(-\widetilde{\tau}m\beta+dn\delta+m\beta\delta-\widetilde{\tau}dn-\widetilde{a}_i dn\gamma+bkm\gamma-\widetilde{a}_i m\beta\gamma-km\alpha\gamma-\widetilde{\tau}k^2m\gamma+\widetilde{q}_i km\beta\gamma\right)\frac{n}{\gamma}\\
&\quad +\frac{K^{tax}}{\gamma}\left(\varphi\left(m\beta+dn\right)+kn\theta\gamma-km\gamma\eta\right)
\end{aligned}$$

with $K^{tax}=\frac{1}{m\beta+dn}$.

Proof of Lemma 1: Suppose $\widetilde{\tau}$ in equation (3) is set equal to the expected price on emissions under ETS:

$$E(\tau^{ets})=\frac{dn^2\delta-Sdn\gamma-Sm\beta\gamma+mn\beta\delta-Adn^2\gamma+bkmn\gamma-Amn\beta\gamma-kmn\alpha\gamma+\widetilde{q}kmn\beta\gamma}{m\gamma k^2n+dn^2+m\beta n}=\overline{\tau}.$$

Then we have the following:

$$\begin{aligned}
p_{\overline{\tau}}^{hyb} &= \frac{bm\beta + dn\alpha - Sdk\gamma - \widetilde{q}dn\beta + dkn\delta + bk^2m\gamma - Adkn\gamma}{m\gamma k^2 + m\beta + dn} + \Psi_p^{hyb} \\
&= \overline{p} + \Psi_p^{hyb} \\
\tau_{\overline{\tau}}^{hyb} &= -\frac{Sdn\gamma - dn^2\delta + Sm\beta\gamma - mn\beta\delta + Adn^2\gamma - bkmn\gamma + Amn\beta\gamma + kmn\alpha\gamma - \widetilde{q}kmn\beta\gamma}{m\gamma k^2 n + dn^2 + m\beta n} + \Psi_\tau^{hyb} \\
&= \overline{\tau} + \Psi_\tau^{hyb} \\
p_{\overline{\tau}}^{tax} &= \frac{bm\beta + dn\alpha - \widetilde{q}dn\beta + (\overline{\tau})\, dkn}{m\beta + dn} + \Psi_p^{tax} \\
&= \frac{bm\beta + dn\alpha - Sdk\gamma - Qdn\beta + dkn\delta + bk^2m\gamma - Adkn\gamma}{m\gamma k^2 + m\beta + dn} + \Psi_p^{tax} = \overline{p} + \Psi_p^{tax} \\
\tau_{\overline{\tau}}^{tax} &= \overline{\tau}
\end{aligned}$$

Hence, expected prices on production and emissions are equal under ETS, HYB and TAX. It follows that the firms expected production and emission levels are equal under the regulatory regimes (given equal technology). Furthermore, we have:

$$\begin{aligned}
p_{\overline{\tau}}^{hyb}(v &= 0) = \overline{p} + \Psi_p^{hyb}(v=0) = \overline{p} + \frac{dkn\varphi + d\eta\,(n + 0\gamma) + \theta\left(n\beta + 0\beta\gamma + k^2n\gamma\right)}{dn^2 + mn\beta + dn0\gamma + m0\beta\gamma + k^2mn\gamma} = \overline{p} + \Psi_p^{ets} \\
\tau_{\overline{\tau}}^{hyb}(v &= 0) = \overline{\tau} + \Psi_\tau^{hyb}(v=0) = \overline{\tau} + \frac{\varphi\,(m\beta + dn) - km\gamma\eta + kn\theta\gamma}{dn^2 + mn\beta + dn0\gamma + m0\beta\gamma + k^2mn\gamma} = \overline{\tau} + \Psi_\tau^{ets} \\
\lim_{v\to\infty}\left(p_{\overline{\tau}}^{hyb}\right) &= \lim_{v\to\infty}\left(\overline{p} + \Psi_p^{hyb}\right) = \overline{p} + \frac{d\gamma\eta + \theta\beta\gamma}{dn\gamma + m\beta\gamma} = \overline{p} + \Psi_p^{tax} \\
\lim_{v\to\infty}\left(\tau_{\overline{\tau}}^{hyb}\right) &= \lim_{v\to\infty}\left(\overline{\tau} + \Psi_\tau^{hyb}\right) = \overline{\tau} + 0 = \overline{\tau}
\end{aligned}$$

Hence, HYB converges towards a tax as $v \to \infty$, and equals ETS if $v = 0$ (given the same technology). In addition, we have

$$\frac{dv}{dp^{hyb}} = \frac{d}{dv}\left(\overline{p} + \Psi_p^{hyb}\right) = -dkn\gamma\frac{dn\varphi + m\beta\varphi + kn\theta\gamma - km\gamma\eta}{\left(dn^2 + mn\beta + dnv\gamma + mv\beta\gamma + k^2mn\gamma\right)^2},$$

$$\frac{dv}{d\tau^{hyb}} = \frac{d}{dv}\left(\overline{\tau} + \Psi_\tau^{hyb}\right) = \frac{-\left(dn\gamma + m\beta\gamma\right)\left(dn\varphi + m\beta\varphi + kn\theta\gamma - km\gamma\eta\right)}{\left(dn^2 + mn\beta + dnv\gamma + mv\beta\gamma + k^2mn\gamma\right)^2}.$$

Neither of which change sign in v. Hence, the equillibrium product price and emissions price under HYB are both monotonic in v. This proves Lemma 1.

Proof of Lemma 2: The lemma follows from the stochastic elements in the reduced form solutions under the regimes above. Specifically, we observe that aggregate production depends on all three stochastic elements under ETS and HYB, but only θ and η under a tax. More precisiely, we have:

$$\begin{pmatrix} \frac{dq^{ets}}{d\varphi} & \frac{dq^{hyb}}{d\varphi} & \frac{dq^{tax}}{d\varphi} \\ \frac{dq^{ets}}{d\eta} & \frac{dq^{hyb}}{d\eta} & \frac{dq^{tax}}{d\eta} \\ \frac{dq^{ets}}{d\theta} & \frac{dq^{hyb}}{d\theta} & \frac{dq^{tax}}{d\theta} \end{pmatrix} = \begin{pmatrix} -mkK^{ets} & -K^{hyb}kmn & 0 \\ -mK^{ets} & -K^{hyb}m\left(n+v\gamma\right) & -mK^{tax} \\ nK^{ets} & K^{hyb}n\left(n+v\gamma\right) & nK^{tax} \end{pmatrix}.$$

Remember that $K^{ets} < K^{tax}$. It is straightforward to compare the expressions to verify that the derivatives under HYB are in between those of ETS and TAX. Lemma 2 follows.

Solutions for production and abatement for individual firms under ETS and TAX when emissions tax equals expected price on emissions under ETS:

$$\begin{aligned} q_{i,\overline{\tau}}^{ets} &= -\frac{1}{\beta}\left(\alpha - p_{\overline{\tau}}^{ets} + \eta_i + k\tau^{ets} - \beta\widetilde{q}_i\right) \\ &= \frac{bmn - mn\alpha + Skm\gamma + \widetilde{q}_i mn\beta - kmn\delta + \widetilde{a}_i kmn\gamma}{m\gamma k^2 n + dn^2 + m\beta n} \\ &\quad + \frac{\left(dn + k^2 m\gamma\right)\left(\eta - n\eta_i\right) - mn\beta\eta_i - km\beta\varphi + n\theta\beta}{m\gamma k^2 n\beta + dn^2\beta + mn\beta^2} \\ &= \overline{q}_i + \Psi_{q_i}^{ets} \end{aligned}$$

$$\begin{aligned} a_i^{ets} &= \frac{1}{\gamma}\left(\tau^{ets} - \delta - \varphi_i + \gamma\widetilde{a}_i\right) \\ &= \frac{-Sm\beta - Sdn - kmn\alpha - k^2 mn\delta + bkmn + \widetilde{q}_i kmn\beta + \widetilde{a}_i k^2 mn\gamma}{m\gamma k^2 n + dn^2 + m\beta n} \\ &\quad + \frac{-km\gamma\eta + \left(m\beta + dn\right)\left(\varphi - n\varphi_i\right) - k^2 mn\gamma\varphi_i + kn\theta\gamma}{mk^2 n\gamma^2 + dn^2\gamma + m\beta n\gamma} \\ &= \overline{a}_i + \Psi_{a_i}^{ets} \end{aligned}$$

$$\begin{aligned} q_{i,\overline{\tau}}^{tax} &= -\frac{1}{\beta}\left(\alpha - p_{\overline{\tau}}^{tax} + \eta_i + k\overline{\tau} - \beta\widetilde{q}_i\right) \\ &= \frac{bmn - mn\alpha + Skm\gamma + \widetilde{q}_i mn\beta - kmn\delta + \widetilde{a}_i kmn\gamma}{m\gamma k^2 n + dn^2 + m\beta n} + \frac{+\theta\beta}{\beta\left(m\beta + dn\right)} - \frac{1}{\beta}\eta_i \\ &= \overline{q}_i + \Psi_{q_i}^{tax} \end{aligned}$$

$$
\begin{aligned}
a_i^{tax} &= \frac{1}{\gamma}\left(\overline{\tau}-\delta-\varphi_i+\gamma A\right)\\
&= -\frac{Sm\beta+Sdn+kmn\alpha+k^2mn\delta-bkmn-\widetilde{q}_ikmn\beta-\widetilde{a}_ik^2mn\gamma}{m\gamma k^2n+dn^2+m\beta n}-\frac{1}{\gamma}\varphi_i\\
&= \overline{a}_i+\Psi_{a_i}^{tax}
\end{aligned}
$$

The letters with an overbar refer to the expected values, while the stochastic elements are represented by the Ψ's.

Variances for production and emissions for individual firms when emissions tax (TAX) equals emissions price under ETS: Define $A_1=\frac{1}{(mk^2n\gamma^2+dn^2\gamma+m\beta n\gamma)^2}$ and $A_2=\frac{1}{\left(m\gamma k^2n\beta+dn^2\beta+mn\beta^2\right)^2}$. Then we have:

$$
\begin{aligned}
var(a_{i,\overline{\tau}}^{ets}) &= A_1\left(\begin{array}{c}\left(-km\gamma\right)^2\sigma_\eta^2+\left(kn\gamma\right)^2\sigma_\theta^2\\ +n\left(\left(dn+m\beta\right)^2+2\left(dn+m\beta\right)\left(-dn-m\beta-k^2m\gamma\right)+n\left(-dn-m\beta-k^2m\gamma\right)^2\right)\sigma_{\varphi_i}^2\end{array}\right),\\
var(q_i^{ets}) &= A_2\left(\begin{array}{c}\left(dn+k^2m\gamma\right)^2\sigma_\eta^2+\left(km\beta\right)^2\sigma_\varphi^2+\left(n\beta\right)^2\sigma_\theta^2\\ +\left(\left(n\left(m\beta+dn+k^2m\gamma\right)\right)^2-2n\left(dn+k^2m\gamma\right)\left(m\beta+dn+k^2m\gamma\right)\left(1+\left(n-1\right)\rho\right)\right)\sigma_{\eta_i}^2\end{array}\right)\\
var(a_{i,\overline{\tau}}^{tax}) &= var\left(\frac{1}{\gamma}\varphi_i\right)=\frac{1}{\gamma^2}\sigma_{\varphi_i}^2,\\
var(q_{i,\overline{\tau}}^{tax}) &= \left(\frac{\beta}{m\beta^2+dn\beta}\right)^2\sigma_\theta^2+\left(\frac{1}{\beta^2\left(m\beta+dn\right)^2}\left(m^2\beta^2+d^2n^2+dm\beta\left(2n+\rho\left(1-n\right)-1\right)\right)\right)\sigma_{\eta_i}^2.
\end{aligned}
$$

Proof of Lemma 3: We first observe that $\sigma_\eta^2=n(1+(n-1)\rho_\eta)\sigma_{\eta_i}^2>0$ iff $\rho_\eta>\frac{-1}{n-1}$, and similar for the other stochastic variables.

Case (i), only $\sigma_\eta^2>0$: Consider the case with $\sigma_\varphi^2=\sigma_\theta^2=0$. Then the derivative of the variance in production under ETS is given by:
$\frac{d\left(var(q_i^{ets})\right)}{d\rho_\eta}=-n\sigma_{\eta_i}^2\left(m\gamma k^2+dn\right)\left(n-1\right)\frac{m\gamma k^2+2m\beta+dn}{\left(m\gamma k^2n\beta+dn^2\beta+mn\beta^2\right)^2}<0$. We further observe that when $\rho=-1/(n-1)$ we have $var(q_i^{ets})=\frac{1}{\beta^2}\sigma_{\eta_i}^2$, whereas $\rho=1$ yields $var(q_i^{ets})=m^2\frac{\sigma_{\eta_i}^2}{(m\gamma k^2+m\beta+dn)^2}$. How does this compare with a tax? The variance in production under TAX when $\rho=-1/(n-1)$ is given by $var(q_i^{tax})=\frac{1}{\beta^2}\sigma_{\eta_i}^2$, whereas $\rho=1$ yields $var(q_i^{tax})=\frac{1}{\beta^2}\frac{\sigma_{\eta_i}^2}{(m\beta+dn)^2}\left(d^2n^2+dmn\beta+m^2\beta^2\right)$. We further have $\frac{d\left(var(q_i^{tax})\right)}{d\rho_\eta}=-d\frac{m}{\beta}\frac{\sigma_{\eta_i}^2}{(m\beta+dn)^2}\left(n-1\right)<0$. It can be shown that $\frac{d\left(var(q_i^{ets})\right)}{d\rho_\eta}-\frac{d\left(var(q_i^{tax})\right)}{d\rho_\eta}<0$. Hence, the variance declines faster in ρ under

ETS. Conclusion case (i): Suppose $\sigma_\varphi^2 = \sigma_\theta^2 = 0$. Then we have $var(q_i^{ets}) \leq var(q_i^{tax})$, with $var(q_i^{ets}) = var(q_i^{tax})$ if $\rho = -1/(n-1)$ and $var(q_i^{ets}) < var(q_i^{tax})$ if $\rho > -1/(n-1)$.

Case (ii), only $\sigma_\varphi^2 > 0$: Consider the case with $\sigma_\eta^2 = \sigma_\theta^2 = 0$. Then we have $\Psi_{q_i}^{ets} = \frac{-km\beta\varphi}{m\gamma k^2 n\beta + dn^2\beta + mn\beta^2}$ and $\Psi_{q_i}^{tax} = 0$. Hence, $var\left(\Psi_{q_i}^{ets}\right) > var\left(\Psi_{q_i}^{tax}\right)$ iff $\rho_\eta > \frac{-1}{n-1}$ for finite n

Case (iii), only $\sigma_\theta^2 > 0$: Consider the case with $\sigma_\varphi^2 = \sigma_\eta^2 = 0$. Then we have $\Psi_{q_i}^{ets} = \frac{n\theta\beta}{m\gamma k^2 n\beta + dn^2\beta + mn\beta^2}$ and $\Psi_{q_i}^{tax} = \frac{\theta\beta}{\beta(m\beta + dn)} = 0$. Hence, $var\left(\Psi_{q_i}^{ets}\right) = \left(\frac{n\beta}{m\gamma k^2 n\beta + dn^2\beta + mn\beta^2}\right)^2 \sigma_\theta^2$ and $var\left(\Psi_{q_i}^{tax}\right) = \left(\frac{\beta}{\beta(m\beta + dn)}\right)^2 \sigma_\theta^2$, which yields $var\left(\Psi_{q_i}^{ets}\right) - var\left(\Psi_{q_i}^{tax}\right) = -k^2 m \frac{\gamma}{(m\beta + dn)(m\gamma k^2 + m\beta + dn)} < 0$ for finite m and n.

The results that HYB is in between follows from Lemma 1. Lemma 3 follows.

Stage 1: The optimial v

We first remember the firms focs in stage 2: ,$-\kappa_{\alpha_i} = E\left(q_i\right)$, $-\kappa_{\beta_i} = \frac{1}{2}E\left(q_i^2\right)$, $-\kappa_{\delta_i} = E\left(a_i\right)$ and $-\kappa_{\gamma_i} = \frac{1}{2}E\left(a_i^2\right)$; and focs of firms and consumers in stage 3: $\frac{d\pi_i}{dq_i} = p - \alpha - \eta_i - k\tau - \beta q_i = 0$,. $\frac{d\pi_i}{dq_i} = \tau - \varphi_i - \gamma a_i = 0$, $\frac{du_j}{dq_i} = b + \theta_j - dq_j - p = 0$.

The social planner solves:

$$W = \max_v E\left[\sum_{j\in J} u(\cdot) - \sum_{i\in I}\left(c\left(\cdot\right) - \kappa\left(\mathbf{x}\right)\right) - g\left(\varepsilon\right) + X\right].$$

With $X = p\left(\sum_{i\in I} q_i - \sum_{j\in J} q_j\right) + \tau\sum_{i\in I}\left(kq_i - kq_i + a_i - a_i\right) = 0$ and subject to $v \geq 0$. The Lagrangian is:

$$L = E\left[\sum_{j\in J} u(\cdot) - \sum_{i\in I}\left(c\left(\cdot\right) - \kappa\left(\mathbf{x}\right)\right) - g\left(\varepsilon\right) + X\right] - \lambda v.$$

Maximization of L requires the standards first and second order conditions,and the complementary slackness conditions (csc):

$$\lambda \geq 0, \text{ with } \lambda = 0 \text{ if } v > 0$$

Note that the csc implies that $\lambda = 0$ if $v > 0$, and that we must have $v = 0$ if $\lambda > 0$ (this does not exclude $\lambda = v = 0$ which is allowed). Differentiating the Lagrangian w.r.t v we get:

$\frac{dL}{dv} = E\left[Y_v\right]$, with Y_v given by:

$$
\begin{aligned}
Y_v &= \sum_{j\in J}(b+\theta_j-dq_j)\frac{dq_j}{dv}-\sum_{i\in I}(\alpha+\eta_i+\beta q_i)\frac{dq_i}{dv}-\sum_{i\in I}q_i\frac{d\alpha}{dv}-\sum_{i\in I}\frac{1}{2}q_i^2\frac{d\beta}{dv}-(\delta+\varphi_i+\gamma a_i)\frac{da_i}{dv}\\
&\quad -\sum_{i\in I}\frac{1}{2}a_i^2\frac{d\gamma}{dv}-\sum_{i\in I}\kappa_\alpha\frac{d\alpha}{dv}-\sum_{i\in I}\kappa_\beta\frac{d\beta}{dv}-\sum_{i\in I}\kappa_\gamma\frac{d\gamma}{dv}-g_\varepsilon\frac{d\varepsilon}{dv}+\frac{dX}{dv}\\
&= \sum_{j\in J}(b+\theta_j-dq_j)\frac{dq_j}{dv}-\sum_{i\in I}(\alpha+\eta_i+\beta q_i)\frac{dq_i}{dv}-\sum_{i\in I}(\delta+\varphi_i+\gamma a_i)\frac{da_i}{dv}\\
&\quad -\sum_{i\in I}(0)\frac{d\alpha}{dv}-\sum_{i\in I}(0)\frac{d\beta}{dv}-\sum_{i\in I}(0)\frac{d\gamma}{dv}-g_\varepsilon\frac{d\varepsilon}{dv}+\frac{dX}{dv}\\
&= \sum_{j\in J}(b+\theta_j-dq_j)\frac{dq_j}{dv}-\sum_{i\in I}(\alpha+\eta_i+\beta q_i)\frac{dq_i}{dv}-\sum_{i\in I}(\delta+\varphi_i+\gamma a_i)\frac{da_i}{dv}-g_\varepsilon\frac{d\varepsilon}{dv}+\frac{dX}{dv}\\
&= \sum_{j\in J}(b+\theta_j-dq_j)\frac{dq_j}{dv}-\sum_{i\in I}(\alpha+\eta_i+\beta q_i)\frac{dq_i}{dv}-\sum_{i\in I}(\delta+\varphi_i+\gamma a_i)\frac{da_i}{dv}-g_\varepsilon\frac{d\varepsilon}{dv}+p\sum_{i\in I}\frac{dq_i}{dv}\\
&\quad -p\sum_{j\in J}\frac{dq_j}{dv}+\tau k\sum_{i\in I}\frac{dq_i}{dv}-\tau k\sum_{i\in I}\frac{dq_i}{dv}+\tau\sum_{i\in I}\frac{da_i}{dv}-\tau\sum_{i\in I}\frac{da_i}{dv}\\
&= \sum_{j\in J}(b+\theta_j-dq_j)\frac{dq_j}{dv}-p\sum_{j\in J}\frac{dq_j}{dv}-\sum_{i\in I}(\alpha+\eta_i+\beta q_i)\frac{dq_i}{dv}+p\sum_{i\in I}\frac{dq_i}{dv}-\sum_{i\in I}(\delta+\varphi_i+\gamma a_i)\frac{da_i}{dv}\\
&\quad +\tau\sum_{i\in I}\frac{da_i}{dv}-g_\varepsilon\frac{d\varepsilon}{dv}+\tau k\sum_{i\in I}\frac{dq_i}{dv}-\tau k\sum_{i\in I}\frac{dq_i}{dv}-\tau\sum_{i\in I}\frac{da_i}{dv}\\
&= -g_\varepsilon\frac{d\varepsilon}{dv}+\tau k\sum_{i\in I}\frac{dq_i}{dv}-\tau k\sum_{i\in I}\frac{dq_i}{dv}-\tau\sum_{i\in I}\frac{da_i}{dv}\\
&= -g_\varepsilon\frac{d\varepsilon}{dv}-\tau\frac{da}{dv}.
\end{aligned}
$$

Hence, the foc is

$$
E\left[-g_\varepsilon\frac{d\varepsilon}{dv}-\tau\frac{da}{dv}\right]=E\left[\tau\left(\frac{d\varepsilon}{dv}-k\frac{dq}{dv}\right)-g_\varepsilon\frac{d\varepsilon}{dv}\right]=\lambda,
$$

with $\lambda\geq 0$, with $\lambda=0$ if $v>0$. This yields equation (14).

Using $a=kq-\varepsilon$, it can be shwown that:

$$
a^{hyb}=-K^{hyb}\left(\begin{array}{c} Sdn^2-kn^2\theta+k^2mn^2\delta+Smn\beta+kmn\eta+dnv\varphi+mv\beta\varphi-Tdn^2v-bkmn^2+kmn^2\alpha\\ +dn^2v\delta+k^2mn\varphi-Ak^2mn^2\gamma-Tmnv\beta+mnv\beta\delta-Adn^2v\gamma-Qkmn^2\beta-Amnv\beta\gamma \end{array}\right).
$$

Define $X=n\left(K^{hyb}\right)^2\left(\begin{array}{c} Tdn^2-m\beta\varphi-dn\varphi-dn^2\delta+Sdn\gamma+Tmn\beta+Sm\beta\gamma-kn\theta\gamma\\ +km\gamma\eta-mn\beta\delta+Adn^2\gamma-bkmn\gamma+Amn\beta\gamma+kmn\alpha\gamma+Tk^2mn\gamma-Qkmn\beta\gamma \end{array}\right).$

Then we have $\frac{d\varepsilon^{byb}}{dv} = -km\gamma X$ and $\frac{da^{byb}}{dv} = (m\beta + dn) X$. It follows that the two terms inside the expecation in equation (14) have opposite signs.

$$\left(dn^2 + mn\beta + dnv\gamma + mv\beta\gamma + k^2mn\gamma\right)^2$$
$$K^{hyb} = 1/\left(dn^2 + mn\beta + dnv\gamma + mv\beta\gamma + k^2mn\gamma\right)$$

$$\left(dn^2 + mn\beta + dnv\gamma + mv\beta\gamma + k^2mn\gamma\right)^2 - \left(dn^2 + mn\beta + dnv\gamma + mv\beta\gamma + k^2mn\gamma\right)^2 = 0$$

Stage 2: Social planner investment: We first note that the first-order conditions (focs) in stage 3 are given by $\frac{d\pi_i}{dq_i} = p - \alpha - \eta_i - k\tau + \beta(Q_i - q_i) = 0$, $\frac{d\pi_i}{da_i} = \tau - \delta_i - \varphi_i + \gamma(A_i - a_i) = 0$, and $\frac{du_j}{dq_i} = b + \theta_j - dq_j - p = 0$. The social planner maximizes welfare:

$$W = \max_{\mathbf{x}} E\left[\sum_{j\in J} u(\cdot) - \sum_{i\in I}(c(\cdot) - \kappa(\mathbf{x})) - g(\varepsilon) + X\right],$$

with $X = p\left(\sum_{i\in I} q_i - \sum_{j\in J} q_j\right) + \tau\sum_{i\in I}(kq_i - kq_i + a_i - a_i) = 0$. Differentiating w.r.t α we get $\frac{dW}{d\alpha} = E[Y_\alpha]$, with Y_α given by:

$$
\begin{aligned}
Y_\alpha &= \sum_{j\in J}(b+\theta_j-dq_j)\frac{dq_j}{d\alpha}-\sum_{i\in I}(\alpha+\eta_i+\beta(q_i-\widetilde{q}))\frac{dq_i}{d\alpha}-\sum_{i\in I}q_i-(\delta+\varphi_i+\gamma(a_i-\widetilde{a}))\frac{da_i}{d\alpha}\\
&\quad -\sum_{i\in I}\kappa_\alpha-g_\varepsilon\frac{d\varepsilon}{d\alpha}+\frac{dX}{d\alpha}\\
&= \sum_{j\in J}(b+\theta_j-dq_j)\frac{dq_j}{d\alpha}-\sum_{i\in I}(\alpha+\eta_i+\beta(q_i-\widetilde{q}))\frac{dq_i}{d\alpha}-\sum_{i\in I}q_i-\sum_{i\in I}(\delta+\varphi_i+\gamma(a_i-\widetilde{a}))\frac{da_i}{d\alpha}\\
&\quad -\sum_{i\in I}\kappa_\alpha-g_\varepsilon\frac{d\varepsilon}{d\alpha}+p\sum_{i\in I}\frac{dq_i}{d\alpha}-p\sum_{j\in J}\frac{dq_j}{d\alpha}+\tau k\sum_{i\in I}\frac{dq_i}{d\alpha}-\tau k\sum_{i\in I}\frac{dq_i}{d\alpha}+\tau\sum_{i\in I}\frac{da_i}{d\alpha}-\tau\sum_{i\in I}\frac{da_i}{d\alpha}\\
&= \sum_{j\in J}(b+\theta_j-dq_j-p)\frac{dq_j}{d\alpha}+\sum_{i\in I}(p-\alpha-\eta_i-\beta(q_i-\widetilde{q})-\tau k)\frac{dq_i}{d\alpha}-\sum_{i\in I}q_i\\
&\quad +\sum_{i\in I}(\tau-\delta-\varphi_i-\gamma(a_i-\widetilde{a}))\frac{da_i}{d\alpha}-\sum_{i\in I}\kappa_\alpha-g_\varepsilon\frac{d\varepsilon}{d\alpha}+\tau k\sum_{i\in I}\frac{dq_i}{d\alpha}-\tau\sum_{i\in I}\frac{da_i}{d\alpha}\\
&= (0)\frac{dq_j}{d\alpha}+(0)\frac{dq_i}{d\alpha}-\sum_{i\in I}q_i+(0)\frac{da_i}{d\alpha}-\sum_{i\in I}\kappa_\alpha-g_\varepsilon\frac{d\varepsilon}{d\alpha}+\tau k\sum_{i\in I}\frac{dq_i}{d\alpha}-\tau\sum_{i\in I}\frac{da_a}{d\alpha}\\
&= -\sum_{i\in I}q_i-\sum_{i\in I}\kappa_\alpha-g_\varepsilon\frac{d\varepsilon}{d\alpha}+\tau\sum_{i\in I}\left(k\frac{dq_i}{d\alpha}-\frac{da_i}{d\alpha}\right)\\
&= -\sum_{i\in I}q_i-\sum_{i\in I}\kappa_\alpha-g_\varepsilon\frac{d\varepsilon}{d\alpha}+\tau\frac{d\varepsilon}{d\alpha}\\
&= -\sum_{i\in I}q_i-\sum_{i\in I}\kappa_\alpha+(\tau-g_\varepsilon)\frac{d\varepsilon}{d\alpha}.
\end{aligned}
$$

Hence, we have the focs $\frac{dW}{d\alpha}=E\left(-\sum_{i\in I}q_i-\sum_{i\in I}\kappa_\alpha+(\tau-g_\varepsilon)\frac{d\varepsilon}{d\alpha}\right)=0$. For each firm $i\in I$ this yields $-\kappa_\alpha=E(q_i)+(g_\varepsilon-\tau)\frac{d\varepsilon}{d\alpha}$. Differentiating w.r.t β we get $\frac{dW}{d\beta}=E[Y_\beta]$, with Y_β given by:

$$
\begin{aligned}
Y_\beta &= \sum_{j\in J}(b+\theta_j-dq_j)\frac{dq_j}{d\beta}-\sum_{i\in I}(\alpha+\eta_i+\beta(q_i-\widetilde{q}))\frac{dq_i}{d\beta}-\frac{1}{2}\sum_{i\in I}(\widetilde{q}-q_i)^2\\
&\quad -\sum_{i\in I}(\delta+\varphi_i+\gamma(a_i-\widetilde{a}))\frac{da_i}{d\beta}-\sum_{i\in I}\kappa_\beta-g_\varepsilon\frac{d\varepsilon}{d\beta}+\frac{dX}{d\beta}.
\end{aligned}
$$

Repeating the steps above, we get $Y_\beta=-\frac{1}{2}\sum_{i\in I}(q_i-\widetilde{q})^2-\sum_{i\in I}\kappa_\beta+(\tau-g_\varepsilon)\frac{d\varepsilon}{d\beta}$, which, for each firm $i\in I$, yields $-\kappa_\beta=\frac{1}{2}E\left((q_i-\widetilde{q})^2\right)+E\left(\frac{g_\varepsilon-\tau}{n}\frac{d\varepsilon}{d\beta}\right)$. Differentiating w.r.t $\widetilde{q}_i$ we get $\frac{dW}{d\widetilde{q}_i}=E\left[Y_{\widetilde{q}_i}\right]$, with $Y_{\widetilde{q}_i}$ given by:

$$
\begin{aligned}
Y_{\widetilde{q}} &= \sum_{j\in J}(b+\theta_j-dq_j)\frac{dq_j}{d\widetilde{q}}-\sum_{i\in I}(\alpha+\eta_i+\beta(q_i-\widetilde{q}))\frac{dq_i}{d\widetilde{q}}-\beta\sum_{i\in I}(\widetilde{q}-q_i)\\
&\quad -\sum_{i\in I}(\delta+\varphi_i+\gamma(a_i-\widetilde{a}))\frac{da_i}{d\widetilde{q}}-\sum_{i\in I}\kappa_{\widetilde{q}}-g_\varepsilon\frac{d\varepsilon}{d\widetilde{q}}+\frac{dX}{d\widetilde{q}}.
\end{aligned}
$$

Repeating the steps above, we get $Y_{\widetilde{q}} = -\beta\sum_{i\in I}(\widetilde{q}-q_i)-\sum_{i\in I}\kappa_\beta+(\tau-g_\varepsilon)\frac{d\varepsilon}{d\beta}$ which, for each firm $i\in I$, yields $-\kappa_\beta = \beta(\widetilde{q}-q_i)+\frac{(g_\varepsilon-\tau)}{n}\frac{d\varepsilon}{d\beta}$. Applying the same procedure for δ and γ and summarizing results yields:

$$
\begin{aligned}
-\kappa_{\alpha_i} &= E(q_i)+E\left(\frac{g_\varepsilon-\tau}{n}\frac{d\varepsilon}{d\alpha}\right),\\
-\kappa_{\beta_i} &= \frac{1}{2}E\left((q_i-\widetilde{q}_i)^2\right)+E\left(\frac{g_\varepsilon-\tau}{n}\frac{d\varepsilon}{d\beta}\right),\\
-\kappa_{\widetilde{q}_i} &= \beta_i E(q_i-\widetilde{q}_i)+E\left(\frac{g_\varepsilon-\tau}{n}\frac{d\varepsilon}{d\widetilde{q}_i}\right),\\
-\kappa_{\delta_i} &= E(a_i)+E\left(\frac{g_\varepsilon-\tau}{n}\frac{d\varepsilon}{d\delta_i}\right),\\
-\kappa_{\gamma_i} &= \frac{1}{2}E\left((a_i-\widetilde{a})^2\right)+E\left(\frac{g_\varepsilon-\tau}{n}\frac{d\varepsilon}{d\gamma_i}\right),\\
-\kappa_{\widetilde{a}_i} &= \gamma_i E(a_i-\widetilde{a}_i)+E\left(\frac{g_\varepsilon-\tau}{n}\frac{d\varepsilon}{d\widetilde{a}_i}\right),
\end{aligned}
$$

which is identical to the firms' first order conditions, except for the expectation involving environmental damage (which is not present in the firms' optimization problem). We observe that this term is zero in the case of pure emissions trading (with an exogenously given and binding emissions cap), and optimal policy and linear environmental damage (with $g_\varepsilon = \tau$).

Suppose we have a TAX policy characterized by $E(g_\varepsilon-\overline{\tau}) = 0$ and the quadratic environmental damage function $g(\varepsilon) = \xi_1\varepsilon+\frac{\xi_2}{2}\varepsilon^2$, with $g_\varepsilon = \xi_2\varepsilon$. Then it can be shown that $g_\varepsilon-\overline{\tau} = \frac{\xi_2}{\gamma}\frac{\varphi(m\beta+dn)+kn\theta\gamma-km\gamma\eta}{m\beta+dn}$. It follows that only the stochastic elements of $\frac{d\varepsilon}{dx}$ remains in the expectation $E((g_\varepsilon-\overline{\tau})\frac{d\varepsilon}{dx})$, where x refers to an element in the technology vector $\mathbf{x}$. The reason is that the constant terms disappears when multiplied with the random elemens with zero expectation). We have $\frac{d\varepsilon^{tax}}{d\alpha} = -km\frac{n}{m\beta+dn}$, $\frac{d\varepsilon^{tax}}{d\beta} = km\frac{m\eta-n\theta+mn\alpha+Qdn^2-bmn+Tkmn}{(m\beta+dn)^2}$, $\frac{d\varepsilon^{tax}}{d\gamma} = -\frac{1}{\gamma^2}(\varphi+n\delta-Tn)$, $\frac{d\varepsilon^{tax}}{dQ} = kmn\frac{\beta}{m\beta+dn}$ and $\frac{d\varepsilon^{tax}}{dA} = -n$. Hence, $E((g_\varepsilon-\overline{\tau})\frac{d\varepsilon^{tax}}{d\alpha}) = E((g_\varepsilon-\overline{\tau})\frac{d\varepsilon}{d\widetilde{q}}) = E((g_\varepsilon-\overline{\tau})\frac{d\varepsilon}{d\widetilde{a}}) = 0$, $E((g_\varepsilon-\overline{\tau})\frac{d\varepsilon^{tax}}{d\beta}) = \frac{-k^2m\gamma\xi_2\left(n^2\theta^2+m^2\eta^2\right)}{\gamma d^3n^3+3\gamma d^2mn^2\beta+3\gamma dm^2n\beta^2+\gamma m^3\beta^3} < 0$ and $E((g_\varepsilon-\overline{\tau})\frac{d\varepsilon^{tax}}{d\gamma}) = -\frac{\xi_2}{\gamma^3}\frac{\varphi^2(m\beta+dn)}{m\beta+dn} < 0$. The same procedure under HYB yields $(g_\varepsilon-\tau)\left(\frac{d\varepsilon^{hyb}}{d\beta}\right) =$

$\frac{km\gamma(v\xi_2-1)v}{(dn^2+mn\beta+dnv\gamma+mv\beta\gamma+k^2mn\gamma)^3}\left(kmn\varphi^2\left(2m\beta+dn\right)-km^2\gamma\eta^2\left(n+v\gamma\right)-2kn^2\theta^2\gamma\left(n+v\gamma\right)\right)$, $\left(g_\varepsilon-\tau\right)\left(\frac{d\varepsilon^{hyb}}{d\gamma}\right)=$ $\frac{-v(m\beta+dn)(v\xi_2-1)}{(dn^2+mn\beta+dnv\gamma+mv\beta\gamma+k^2mn\gamma)^3}\left(\varphi^2\left(m\beta+dn\right)\left(mv\beta+k^2mn+dnv\right)-\gamma k^2m^2n\eta^2-\gamma k^2n^3\theta^2\right)$, and $E((g_\varepsilon-\overline{\tau})\frac{d\varepsilon^{hyb}}{d\alpha})=E((g_\varepsilon-\overline{\tau})\frac{d\varepsilon^{hyb}}{d\widetilde{q}})=E((g_\varepsilon-\overline{\tau})\frac{d\varepsilon^{hyb}}{d\widetilde{a}})=0$. Hence, given the above assumptions (i.e., $E\left(g_\varepsilon-\overline{\tau}\right)=0$ and $g_\varepsilon=\xi_2\varepsilon$) we can remove four of the expectations involving environmental damage in the above system of equations, which yields equation (15).